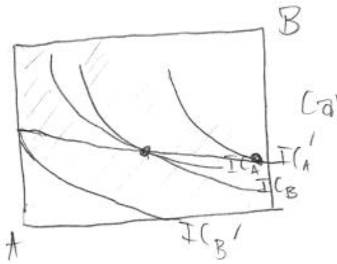
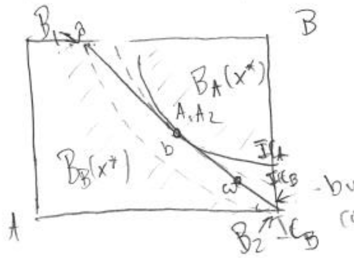
$\exists$ PTE $\Rightarrow$ Replica invariance
$\Leftarrow$ ← This is almost always true.

must separate "better than" sets



cannot decentralize this. assume a price vector exists.
↳ market does not clear,

Suppose we replicate the economy



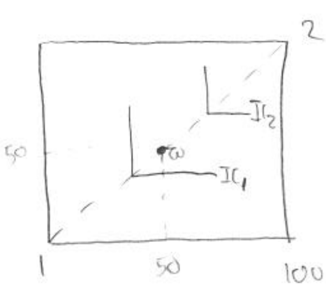
assume $|\overline{ab}| = |\overline{bc}|$

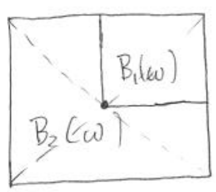
If we replicate the economy once and have B_1, B_2, A_1, A_2

In effect, this convexifies the better than sets for type B consumers.

2003 Question 1

1: $\min\{x,y\}$ $\omega_1 = (50,50)$ "pessimist"
2: $\max\{x,y\}$ $\omega_2 = (50,50)$ "optimist"

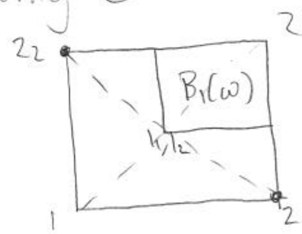


a) 

There is no region of mutually advantageous trade

b) Assume replicated economy $\mathcal{E}^2$:

$1_1 = (50, 50)$
$1_2 = (50, 50)$
$2_1 = (100, 0)$
$2_2 = (0, 100)$

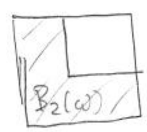


$p = 1$

In $\mathcal{E}$, $u_1 = 30$, $u_2 = 50$
In $\mathcal{E}^2$, $u_1 = 50$, $u_2 = 100$ $\forall 1, \forall 2$.
Clearly <u>not</u> replica invariant

c) There is no price-taking equilibrium in $\mathcal{E}$.
Since $\exists$PTE $\Rightarrow$ Replica invariance, $\neg$Replica invariance $\Rightarrow \nexists$ PTE

d) Convexity of preferences



$B_2(\omega)$ is not convex

e) $\mathcal{J} = \mathcal{E}^2$. Show that $\mathcal{J}$ is replica invariant.

$2n$

$$\left.\begin{array}{l} 1_i \ (50,50) \\ 2_i \ (100,0) \text{ or } (0,100) \\ p=1 \end{array}\right\} \text{optimal}$$

<u>Quasilinear preferences</u>

$u(x, m) = v(x) + m$

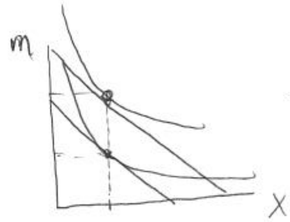
$\max \ v(x) + m \quad \text{s.t.} \quad I - p \cdot x - m = 0$

$\mathcal{L} = v(x) + m + \lambda(I - p \cdot x - m)$

$(x) \quad \nabla v(x) = \lambda p \quad \Rightarrow \quad h = v(x) + I$

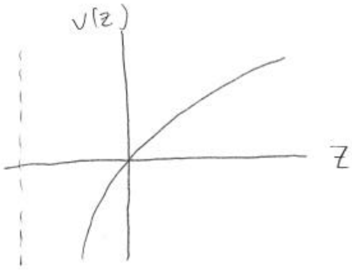
$(m) \quad \lambda = 1$

$\nabla v(x) = p \Rightarrow$ not a function of $I \Rightarrow$ independent of income.

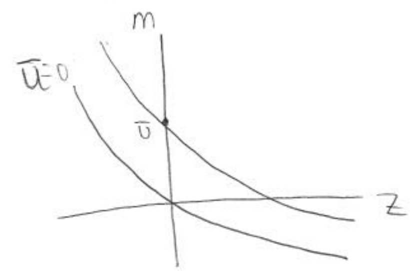


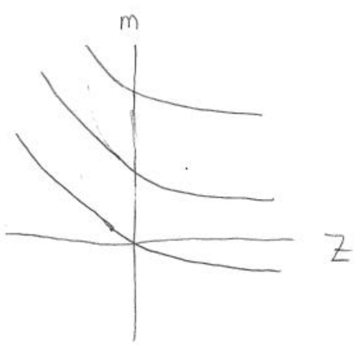
$I \uparrow \Rightarrow m \uparrow$, x constant.

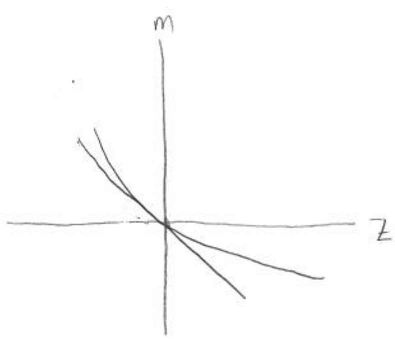
$v(z) + m$, $v(0) = 0$



$\bar{u} = v(z) + m \Rightarrow m = \bar{u} - v(z)$







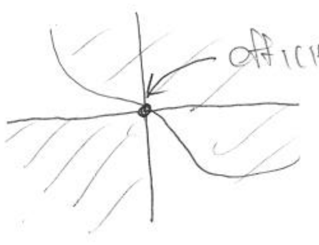
For a one person economy:

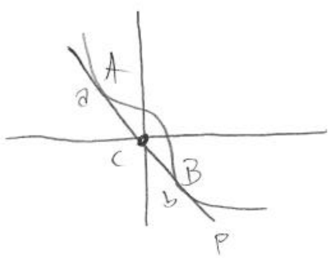
$(z, m) = \arg\max \{ v(z) + m \mid p \cdot z + m = 0 \}$

$= \arg\max \{ v(z) - p \cdot z \}$

$z = 0$, $m = 0$ is the market clearing condition

$v'(0)$ is the market clearing price





$|ac| = 1$
$|bc| = 2$

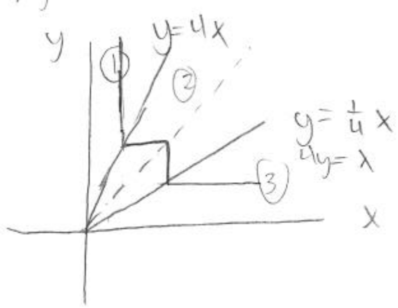
If we triple the economy, it can be decentralized: Two consumers at A and one consumer at B.

$\{A, B\} = \arg\max \{v(z) - p \cdot z\}$

If we have a continuum of consumers, we can "convexify" the strict preference sets.

Comp Fall 2003 Q5

$u(x,y) = \max\{\min\{x, 4y\}, \min\{4x, y\}\}$, $\omega = (\alpha, \beta) \geq 0$



① : $y > 4x$ and $4y > x$

$\Rightarrow u = \max\{4x, x\} = 4x$

③ : $y < 4x$ $4y < x$

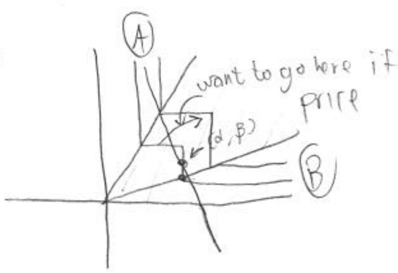
$\Rightarrow u = \max\{y, 4y\} = 4y$

② : $4y > x$ $y < 4x$

$\Rightarrow u = \max\{x, y\}$

a)

b)



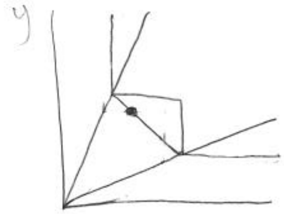
Thus no endowments in //// region can be price-taking equilibrium

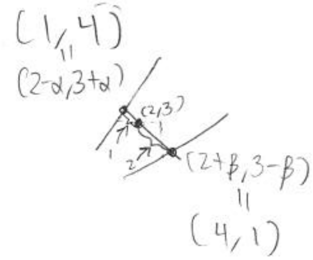
In region Ⓐ, $\frac{p_1}{p_2} = \infty$ supports this

In region Ⓑ, $\frac{p_1}{p_2} = 0$ supports this

On the lines, there are a continuum of equilibria

c) (α, β) is in //// region $\Rightarrow$ No price-taking equilibrium

d) 



$y = 4x$

$3+\alpha = 4(2-\alpha) \Rightarrow \alpha = 1$

$4y = x$

$4(3-\beta) = 2+\beta$

$12 - 4\beta = 2+\beta$

$\beta = 2$

f) $u(x,y) = \begin{cases} 4x & ① \\ c(x+y) & ② \\ 4y & ③ \end{cases}$