① Outline the proofs of the first and second welfare theorems.

First Theorem - Efficiency of price-taking equilibrium (Comp. Eq.) - No convexity required

Second Theorem - Pricing characterization of efficiency - convexity required.

Main conclusion: There is "no difference" between the two theorems. Both are characterizations of efficiency.

② Quasilinear model of general equilibrium

$\mathcal{E}$: private ownership - $w_i(p) = p\omega_i + \sum_j \theta_{ij}\pi_j(p)$

$[(x_i^*), (y_j^*), p]$ $\quad x_i^* \in \xi_i(p, w_i(p))$, $y_j^* \in \eta_j(p) \Rightarrow x_i^* \in X_i$

u-max ① $\quad$ π-max ② $\quad y_j^* \in Y_j$

$$\sum_i x_i^* - \sum_j y_j^* = \omega = \sum \omega_i$$

$$x^* - y^* = \omega \leftarrow \text{mkt clearance ③}$$

$x \in Y + \omega \Rightarrow x = y + \omega$ for some $y \in Y$

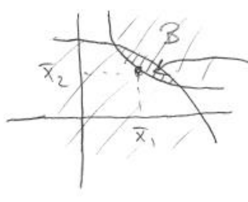
Geometric conditions for efficiency

$[(\bar{x}_i), (\bar{y}_j)]$ is an attainable allocation

$$B[(\bar{x}_i)] = \sum_i B_i(\bar{x}_i) \in \mathbb{R}^l$$

$Y + \omega \subset \mathbb{R}^l$ $\qquad Y = \sum_j Y_j$

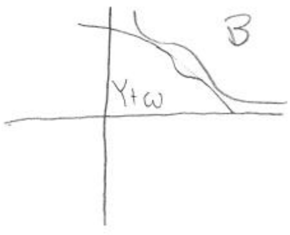
$B[(\bar{x}_i)] \cap \{Y + \omega\} \neq \emptyset \Rightarrow$ Not weakly efficient



If each $B_i(\bar{x}_i)$ is convex, $\sum_i B_i(\bar{x}_i)$ is convex

can find an allocation to make everyone better off.

$B[(\bar{x}_i)] \cap \{Y + \omega\} = \emptyset$ in equilibrium.



$\mathcal{E}$ $(\succsim_i)$ (Y_j)

Suppose we have $\hat{\mathcal{E}}$. $\hat{B}_i(x_i)$ - convex hull of $B_i(x_i)$

$\hat{B}_i(x_i) = \{ b \mid b = \sum_{k=1}^{l+1} \alpha_k b_k \quad \alpha_k \geq 0, \ \sum \alpha_k = 1 \}$ ← result from convex analysis.

$\hat{\mathcal{E}} = [(X_i, \succsim_i), (\hat{Y}_j), \omega, (\theta_{ij})]$ with $\hat{B}_i(x_i)$

Proposition: If $[(x_i^*), (y_j^*), p]$ is a PTE for $\mathcal{E}$, then it is a PTE for $\hat{\mathcal{E}}$

$y_j^* \in \eta_j(p) \longrightarrow \sup pY_j$. Suppose we look at $\sup p\hat{Y}_j$

$\sup p\hat{Y}_j \geq \sup pY_j$

Suppose $\sup p\hat{Y}_j > \sup pY_j$. Then $\exists\, p\hat{y}_j > \sup pY_j$

$y_j = \sum_{k=1}^{l+1} \alpha_k y_{jk}$ where $y_{jk} \in Y_j$

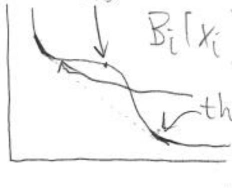
$\Rightarrow p\left(\sum_{k=1}^{l+1} \alpha_k y_{jk}\right) > \sup p\hat{Y}_j > \sup pY_j \Rightarrow \exists_k \ni p y_{jk} > \sup pY_j \rightarrow\leftarrow$

since $\sup pY_j$ is clearly π-max.

Similarly, if $x_i^* \in \xi_i(p, w_i(p))$, then $x_i^* \in B_i(x_i^*)$

$\Rightarrow x_i^* \notin \hat{B}_i(x_i^*)$

this could not possibly be u-max st $B_i(x_i)$ a linear b.c.

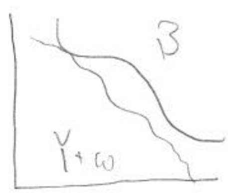


these could be u-max st a linear b.c.

$\Rightarrow$ same results if we convexify.

Second thm. $(x_i), (y_j), \omega$

$B[(x_i)] \cap \{Y + \omega\} = \emptyset$



$\Rightarrow$ no supporting hyperplane.

This holds for weak optimality as stated. If we assume local non-satiation, we get strong optimality.

1st thm depends on $\mathcal{E}$ } Ostroy says neither theorem "depends on property
2nd thm depends on E. } rights"

$$w_i(p) = p\omega_i + \sum_j \theta_{ij}\pi_j(p) \qquad \theta_{ij} \notin \{0,1\}$$

Person i has the right to exclude others from using this.

there is no justification in the model because there is no uncertainty and financial markets work perfectly.

$$w(p) = p\cdot\omega + \sum_j \pi_j(p)$$ ← total wealth at prices p.

$$w_i(p) = \alpha_i\, w(p) \qquad \alpha_i \geq 0, \quad \sum \alpha_i = 1$$

Market socialism: Blend efficiency with equality: $\alpha_i = \frac{1}{N}\ \forall i$

$$\omega_i = \bar{\omega}_i + r_i \xrightarrow{\text{give those to firms}} y_j + r_j \quad \text{where} \quad \sum r_j = \sum r_i$$

inalienable resources; alienable resources

Producers will do their thing: $\sup p(Y_j + r_j)$

$\sup pY_j + pr_j$ → can get efficiency in production

$$\tilde{w}_i(p_i) = \alpha_i w(p) \qquad \alpha_i w(p) \geq p\bar{\omega}_i$$

Suppose an exchange economy: $Y_j = \{0\}\ \forall j$ $\quad \tilde{\mathcal{E}}$

$\Rightarrow w(p) = p\cdot\omega$

$\tilde{w}_i(p) = \alpha_i w(p)$

Quasilinear General Equilibrium: Description of individual characteristics

Revise two asymmetries in the standard model: 1) Consumers (final consumption) (consumers will be described by "trade"); producers (trades); 2) adopt the same sign convention for consumers and producers 3) Eliminate the distinction between producer and consumer.

Quasilinearity will give $\mathbb{R}^{\ell+1}$ and if $z \in \mathbb{R}^\ell$, $m \in \mathbb{R}$ (money commodity), $U(z,m) = v(z) + m$ → no income effects for the first ℓ commodities.