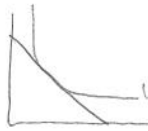
E - economy $\{(X_i, \succsim_i), (Y_j); \omega\}$ ω

$\mathcal{E}$ - Private ownership economy $\sum_i \omega_i = \omega$; $\theta_{ij} \geq 0$, $\sum_i \theta_{ij} = 1 \ \forall j$

$\xi_i(p, w_i)$ - Demand correspondence (utility maximizing bundles at p)



- This is a correspondence because there may be multiple utility maximizing bundles.

$w_i(p) = p \cdot \omega_i + \sum_j \theta_{ij} \pi_j(p)$

Equilibrium for $\mathcal{E}$

$[(x_i^*), (y_j^*), p]$ satisfying

1) $x_i^* \in \xi_i(p, w_i(p))$
2) $y_j^* \in \eta_j(p)$
3) $\sum_i x_i^* - \sum_j y_j^* = \omega$

Criticism: This says too much It is not a good representation of the invisible hand. Will discuss later (It relies too heavily on the assumption of price-taking)

Since no one has any capacity to do anything until prices are known, we will refer to this as a price-taking equilibrium

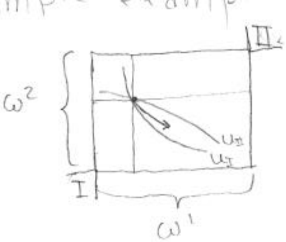
Prices add coherence to the equilibrium In addition, in some sense, the equilibrium is the best that it can possibly be

Pareto optimality

This notion applies to E

① $[(x_i), (y_j)]$, $x_i \in X_i$, $y_j \in Y_j$, $\sum_i x_i - \sum_j y_j = \omega$

Simple example is Edgeworth box



PO ⇒ ① and can't make anyone better off without hurting someone else.

The distinction between weak PO and strong PO

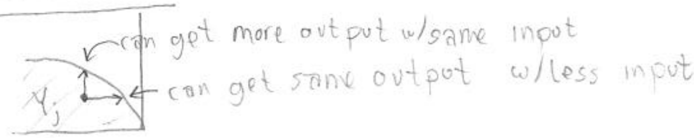
L can't make everybody better off

L See above

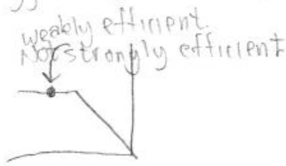
If a state is efficient for E, it is efficient for $\mathcal{E}$.

Efficiency in production

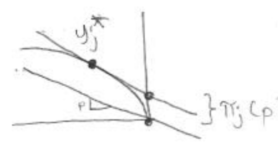
$y_j \in Y_j$



$y_j \in Y_j$ is efficient if $\nexists\, y_j' \in Y_j \ni y_j' \geq y_j$ and $y_j' \neq y_j$



Pricing and Efficiency



$\sup_{y_j \in Y_j} p y_j = \pi_j(p)$, $p \gg 0$

Suppose $y_j^* \in \eta_j(p)$

Proposition: If $p \gg 0$ and $y_j^* \in \eta_j(p)$, then y_j^* is efficient with respect to Y_j.

Pf: Trivial.

Decentralization (wrt production)

$Y = \sum_j Y_j$ where $Y_i + Y_k = \{ y \mid y = y_i + y_k,\ y_i \in Y_i,\ y_k \in Y_k \}$

$Y =$ aggregate input-output feasibility set
$=$ aggregate technology

$\pi(p) = \sup_{y \in Y} p y$ (centralized problem)

$\pi_j(p) = \sup_{y_j \in Y_j} p y_j$ (decentralized problem)

Proposition:
$\pi(p) = \sum_j \pi_j(p)$

$\sup p[Y_1 + Y_2 + \dots + Y_J] = \sup pY_1 + \quad + \sup pY_j$

Suppose $\pi(p) > \Sigma\, \pi_j(p)$

$$\Downarrow$$

$$p\tilde{y} > \sum_{j=1}^{J} p y_j$$

$\tilde{y} = \tilde{y}_1 + \dots + \tilde{y}_J$

$p\tilde{y} = p\tilde{y}_1 + \dots + p\tilde{y}_J \quad \tilde{y}_j \in Y_j$ Then $\exists\, j \ni p\tilde{y}_j > p y_j = \pi_j(p) \rightarrow\!\leftarrow$

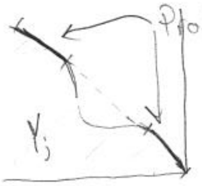
There is no loss in decentralization

There is a tacit assumption: $\frac{dy_{jl}}{dy_{kl}} = 0$ i.e. goods are private. No externalities.

When does efficiency in production imply prices?

- Not always

Does $\exists\, p \ni \pi_j(p) = p\bar{y}_j$? No.



Profit maximization will trace out these segments.

≡ convex hull of Y_j

$S \subset \mathbb{R}^l$ S is convex if $s_1, s_2 \in S$, $\lambda \in (0,1) \Rightarrow \lambda s_1 + (1-\lambda)s_2 \in S$

Convexity of Y_j is a key assumption

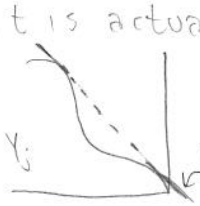
$\hat{Y}_j \equiv$ smallest convex set containing Y_j

$Y_j \subseteq \hat{Y}_j$

$\hat{\pi}_j(p) = \sup p\hat{Y}_j$

We know $\hat{\pi}_j(p) \geq \pi_j(p)$

It is actually true that $\hat{\pi}_j(p) = \pi_j(p)$

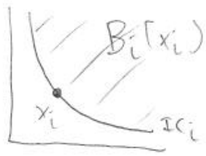


This is π-max for both Y_j and $\hat{Y}_j$

Theorem:
an equilibrium $[(x_i^*), (y_j^*), p]$ for $\mathcal{E}$ is weakly PO for E.

Let $B_i(x_i) = \{x_i' \mid x_i' \succ_i x_i\}$ = set of consumption bundles that are better than x_i. (strictly)

where $x_i' \succ_i x_i \Leftrightarrow x_i' \succsim_i x_i \wedge \neg(x_i \succsim_i x_i')$



$B_i(x_i)$ (not including boundary.

$B[(x_i)] = \sum B_i(x_i)$. $\quad \bar{x} \in B[(x_i)] \Rightarrow \bar{x} = \sum \bar{x}_i$ and $\bar{x}_i \in B_i(x_i)$

$[(x_i), (y_j)]$

$\sum x_i - \sum y_j = \omega$

$\sum x_i = \sum y_j + \omega$ ← attainable allocation

If $[(x_i), (y_i)]$ is attainable, it is weakly PO if

$B[(x_i)] \cap \{Y + \omega\} = \emptyset$ (there is no attainable allocation that can make everybody better off)