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OH: Thursday 12:00-2:00 (2265 Bunche)

Big picture
- PTE (Price-taking equilibrium) ≠ PCE (Perfectly competitive equilibrium)
  - What is PTE for a given economy?
  - What is price?
- Incentives
  - Double auctions, matching, GE
  - Mechanism design (planner's problem)

answer: Perfect competition
Individual marginal productivity
Devote a lot of time to linear programming and standard model.
└ Read lecture notes on linear programming before next week

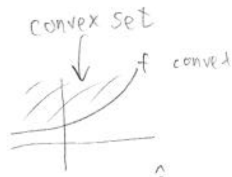
$MP_i = V - V_{-i}$ Typically, $y_i \le MP_i$. In PCE, $y_i = MP_i \; \forall i$

"value" of economy $\{1, \ldots, N\}$ $\{1, \ldots, i-1, i+1, \ldots, N\}$

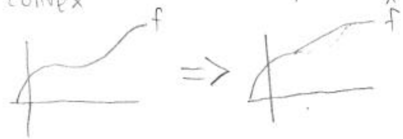
- ω endowment, W wealth
- convexity

not convex

convex



"convexify" function:

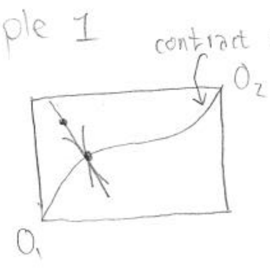


Welfare Theorem
(1) PTE ⇒ PO
(no externalities, ...)
(2) PO ⇒ PTE
(no externalities, convexity

1. no externalities (private goods)
2. price-taking
3. local non-satiation

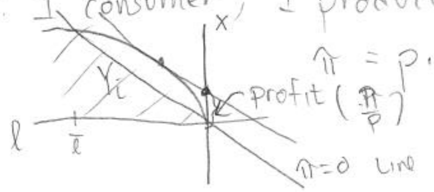
1,2,3 + 4. convexity

Example 1



Given a point on contract curve $\exists\, \omega$ and $\exists\, p \ni$ it is an equilibrium (2nd Welfare Thm)

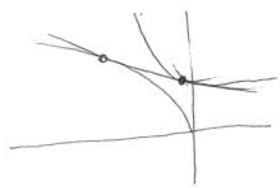
Example 2: 1 consumer; 1 producer economy



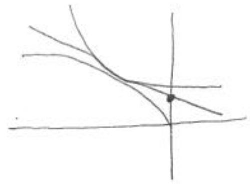
$\pi = p \cdot x - r \cdot l$

$l = 0 \Rightarrow \pi = p \cdot x \Rightarrow x = \frac{\pi}{p}$

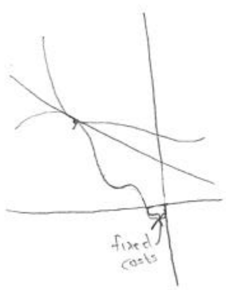
$\max u(x, l)$ st $p \cdot x + r \underbrace{(\bar{l} - l)}_{\text{leisure}} = W + r \cdot \bar{l}$



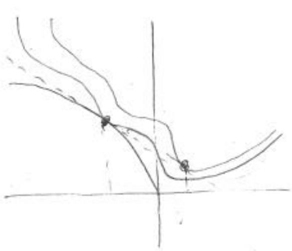
not in equilibrium



$p \cdot x + rl = p \cdot \left(\frac{\pi}{p}\right) = \pi$



We can <u>possibly</u> support an efficient allocation without convexity assumptions.
2^{nd} welfare thm is sufficient condition.
Necessary condition is existence of separating hyperplane.

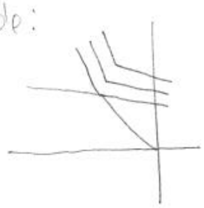


no separating hyperplane.

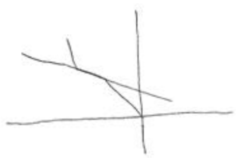
Tripled economy.
3 producers
3 consumers

Then we can support an efficient allocation with prices.

Example:

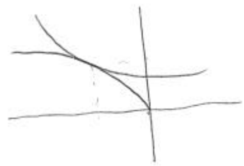


We have a continuum of equilibria here.
Unique allocation though



Unique price line
Continuum of allocations

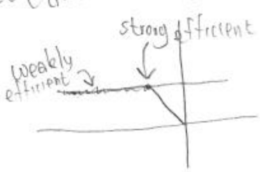
Production efficiency vs. economic efficiency

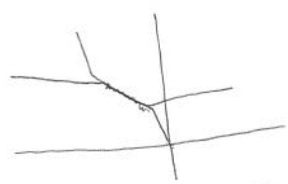


Production efficiency
- must produce on NE side of prod. set

Economic efficiency

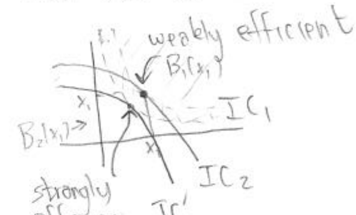
Weak vs. strong





Strong economic efficiency

Local non-satiation



at the weakly efficient point, the separating hyperplane separates $B_1(x_1)$ & $B_2(x_2)$

Under non-satiation, weak efficiency $\Leftrightarrow$ strong efficiency

2003 Homework 1 question 2

2 consumers $\quad u = [y^{-\rho} + \ell^{-\rho}]^{-\frac{1}{\rho}} \quad \begin{cases} \bar{L}_1 = 3, \ \theta = 0 \\ \bar{L}_2 = 0, \ \theta = 1 \end{cases}$

1 producer $\quad y = 2\sqrt{L}$

$P_y = 1$; w = price of labor

Firm $\quad \pi = 2\sqrt{L} - wL$

$\frac{\partial \pi}{\partial L} = 0 \Rightarrow \frac{1}{\sqrt{L}} - \omega = 0 \Rightarrow L^d = \frac{1}{\omega^2} \Rightarrow y^* = \frac{2}{\omega} \Rightarrow \pi^* = \frac{1}{\omega}$

Consumer 1: $MRS = \frac{MU_\ell}{MU_y} = \left(\frac{y}{\ell}\right)^{1+\rho} = \frac{\omega}{1} \Rightarrow y = \omega^{\frac{1}{1+\rho}} \ell$

B.C. $\quad y + \omega \ell \leq 3\omega$

$\Rightarrow (3 - \ell)\, \omega = \omega^{\frac{1}{1+\rho}} \ell$

$\ell^* = \frac{3\omega}{\omega^{\frac{1}{1+\rho}} + \omega}$

$y^* = \frac{3\omega^{\frac{2+\rho}{1+\rho}}}{\omega^{\frac{1}{1+\rho}} + \omega}$

Consumer 2: $\quad y + \omega \ell \leq \pi$

$y + \omega \ell = \frac{1}{\omega}$

$y = \omega^{\frac{1}{1+\rho}} \ell$

$\ell^* = \frac{1/\omega}{\omega^{\frac{1}{1+\rho}} + \omega}$

Mkt clearing: $\quad 3 = \frac{1}{\omega^2} + \frac{3\omega}{\omega + \omega^{\frac{1}{1+\rho}}} + \frac{1/\omega}{\omega + \omega^{1/1+\rho}} \qquad \omega^* = 1$