Distinction between price-taking equilibrium and perfectly competitive equilibrium

1st week: Overview of GE model and why it does not do what it is supposed to

2nd week: GE model with important simplification: quasi-linear preferences

Note: Quasilinear preferences have no income effects.

3rd week: Distinction between price-taking and perfectly competitive.

4th week: Connections between prices and quantities (duality). "prices of individuals"

5th week: Game theoretic connection between correlated equilibrium and NE

Efficiency properties of competitive equilibrium.

6th week: Midterm. Existence of perfectly competitive equilibrium. Requires some degree of substitutability

The logic of "appropriation" will be central to this course.

7th week: Mechanism design, auctions, and appropriability.

Last topic: Two kinds of asymmetric information. Privacy vs. delivery problems.

* Read Hayek "The Use of Knowledge in Society"

 └ This is "privacy" asymmetric information - info. you don't want to know

"Delivery problems" - info I have that you want to know. Market for lemons

 └ Cannot credibly deliver such information.

Markets work well wrt privacy, but not delivery problems

Standard model of general equilibrium

Commodities: ℓ commodities. Commodity bundles are in $\mathbb{R}^\ell$, $1 \le \ell < +\infty$ finite

Prices: Linear functions on commodity space. $p = (p_1, \dots, p_\ell)$

If $x \in \mathbb{R}^\ell$, $p \cdot x = \sum_{k=1}^{\ell} p_k x_k$ where $x = (x_1, \dots, x_\ell)$

Producers: j, a producer is described by a set Y_j, the set of feasible input-output vectors. $y_j \in Y_j$, $y_j = (y_{j1}, \dots, y_{j\ell})$. $y_{jk} < 0 \Rightarrow$ input

$y_{jk} > 0 \Rightarrow$ output

"standard" notation

$p \cdot y_j \equiv$ profit

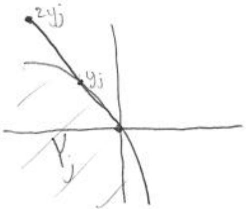
Goal of the producer: $\boxed{\pi_j(p) = \sup_{y_j \in Y_j} p \cdot y_j}$

Ordinarily we draw a prod fcn:

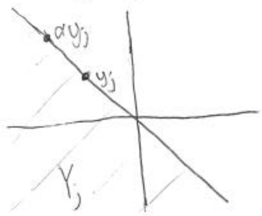
Here, we have (since inputs are negative):

We must write $\pi_j(p) = \sup_{y_j \in Y_j} p y_j$ instead of $\pi_j(p) = \max_{y_j \in Y_j} p y_j$

Suppose there are constant returns to scale: $y_j \in Y_j \Rightarrow \alpha y_j \in Y_j \ \forall \alpha > 0$



$2y_j \notin Y_j \Rightarrow$ Not constant returns to scale



$\alpha y_j \in Y_j \ \forall \alpha > 0 \Rightarrow$ CRS

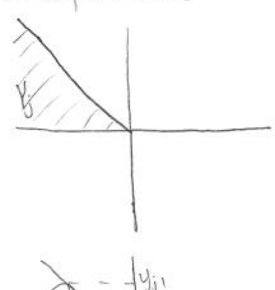
Proprietary vs non-proprietary technology

Non-proprietary - the blueprints are out there. all you have to do is use them

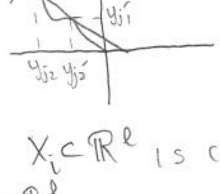
- Can replicate CRS by using the same blueprint over and over
- The return to using a blueprint in equilibrium should be zero

Proprietary view - Each producer has, in some sense, its "own" production set.

Supply correspondence: $\eta_j(p) = \{ y_j \mid p \cdot y_j = \pi_j(p) \}$



If profits are zero, and technology is CRS, the whole frontier is in the supply correspondence.



$\eta_j(p) = \{ (y_{j1}, y_{j2}), (y_{j1}', y_{j2}') \}$

Consumer: i. $X_i \subset \mathbb{R}^\ell$ is called the consumption set in $\mathbb{R}^\ell$

For now, $X_i = \mathbb{R}^\ell_+$

Preferences: $\succsim_i$. Binary relation on $X_i \times X_i$

Suppose $u(x_i) \geq u_i(x_i')$. This is the same as $x_i \succsim_i x_i'$

Budget set: $\gamma_i(p, w_i) = \{ x_i \in X_i \mid p x_i \leq w_i \}$ where w_i is i's "wealth".

Demand correspondence: $\xi_i(p, w_i) = \{ x_i \in X_i \mid x_i \in \gamma_i(p, w_i),\ x_i \succsim_i \gamma_i(p, w_i) \}$

Distinction between economy and private ownership economy.

Economy: $E \equiv \{(X_i, \succsim_i), (Y_j), \omega\}$ where $\omega \in \mathbb{R}^\ell_+$, $\omega \equiv$ aggregate quantities of resources

Private Ownership Economy: $\mathcal{E} \equiv \{(X_i, \succsim_i, \omega_i), (Y_j), (\theta_{ij})\}$ where $\sum_i \omega_i = \omega$

and $\theta_{ij} \geq 0 \ni \sum_i \theta_{ij} = 1 \ \forall j$, $\{\theta_{ij}\} \equiv$ assignment of profit shares.

State of E: $[(x_i), (y_j)]$

attainable state of E: $[(x_i), (y_j)]$, $x_i \in X_i$, $y_j \in Y_j$, $\sum x_i - \sum_j y_j = \omega$

$$\sum_i \begin{bmatrix} x_{i1} \\ \vdots \\ x_{i\ell} \end{bmatrix} - \sum_j \begin{bmatrix} y_{j1} \\ \vdots \\ y_{j\ell} \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_\ell \end{bmatrix} \iff \underbrace{\sum_i \begin{bmatrix} x_{i1} \\ \vdots \\ x_{i\ell} \end{bmatrix}}_{\text{demand}} = \underbrace{\sum_j \begin{bmatrix} y_{j1} + \omega_{j1} \\ \vdots \\ y_{j\ell} + \omega_{j\ell} \end{bmatrix}}_{\text{supply}}$$

$w_i = p\omega_i + \sum_j \theta_{ij} \pi_j(p)$ – Given prices, this is i's wealth.

an equilibrium (price taking) is a vector of allocations and prices $[(x_i^*), (y_j^*), p^*]$ such that

1) $x_i^* \in \xi_i(p^*, w_i)$,
2) $y_j^* \in \eta_j(p^*)$, and
3) $[(x_i^*), (y_j^*)]$ is attainable

Note: We can let $\theta_i = (\theta_{i1}, \ldots, \theta_{iJ})$ and $\pi(p) = (\pi_1(p), \ldots, \pi_J(p))$. Then,

$w_i = p \cdot \omega_i + \theta_i \cdot \pi$