Econ201C: General Equilibrium and Welfare Economics

Table of Contents

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Lecture 1 (4/4/05) - 3 pages

Key points

- Overview of course material
- Standard model
- Returns to scale
- Propriety vs. non-proprietary technology
- Economy vs. private ownership economy
- Price-taking equilibrium

Notation and definitions

- Y_j - production set
- $\pi_j = \sup_j \{p \cdot y_j\}$ - profit function
- X_j - consumption set
- $\eta_j(p) = \{y_j : p \cdot y_j = \pi_j(p)\}$ - supply correspondence
- $\gamma_i(p, w_i(p)) = \{x_i \in X_i : p \cdot x_i \leq w_i(p)\}$ - budget set
- $\xi_i(p, w_i(p)) = \{x_i \in X_i : x_i \in \gamma_i(p, w_i(p)), x_i \succeq_i \gamma_i(p, w_i(p))\}$ - demand correspondence
- ω - endowment
- $E = \{(X_i, \succeq_i), (Y_j), \omega\}$ - economy
- $\varepsilon = \{(X_i, \succeq_i, \omega_i), (Y_j), (\theta_{ij})\}$ - private ownership economy
- $[(x_i), (y_j)]$ s.t. $\sum_i x_i - \sum_j y_j = \omega$ - attainable state of E

Definition 1 *A price taking equilibrium (for the economy ε) is a vector of allocations and prices $\left[(x_i^*), (y_j^*), p^*\right]$ s.t.*

- 1. $x_i^* \in \xi_i(p^*, w_i(p^*))$
  2. $y_j^* \in \eta_j(p^*)$
  3. $\left[(x_i^*), (y_j^*)\right]$ is attainable

Lecture 2 (4/6/05) - 4 pages

Key points

- Pareto optimality
- Weak vs. strong optimality
- Efficiency in production

  Proposition 2 *If $p >> 0$ and $y_j^* \in \eta_j(p)$, then y_j^* is efficient with respect to Y_j*

- Decentralization

  Proposition 3 $\pi(p) = \sum_j \pi_j(p)$

- Assumption of private goods
- Efficiency and prices
- Convexification

  Theorem 4 $\left[(x_i^*), \left(y_j^*\right), p\right]$ *for ε is weakly Pareto optimal for E.*

Notation and definitions

- $w_i(p) \equiv p \cdot \omega_i + \sum_j \theta_{ij} \pi_j(p)$

  Definition 5 *$y_j \in Y_j$ is efficient if $\nexists\, y_j' \in Y_j$ s.t. $y_j' \geq y_j$ and $y_j' \neq y_j$.*

- $Y = \sum_j Y_j = \left\{\sum_j y_j : y_j \in Y_j \; \forall j\right\}$ - aggregate input-output feasbility set
- $\pi(p) = \sup_{y \in Y} p \cdot y$ - centralized problem

  Definition 6 *S is convex if $s_0, s_1 \in S, \lambda \in (0,1) \Rightarrow \lambda s_0 + (1-\lambda) s_1 \in S$*

- $\hat{Y}_j \equiv$ smallest convex set containing Y_j
- $B_i(x_i) \equiv \{x_i' : x_i' \succ_i x_i\}$ - strict preference set
- $B[(x_i)] = \sum_i B(x_i)$

  Definition 7 *If $[(x_i), (y_j)]$ is attainable, it is weakly PO if $B[(x_i)] \cap \{Y + \omega\} = \emptyset$*

Discussion 1 (4/6/05) - 4 pages

Key points

- Big picture
- Welfare theorems
- Production efficiency vs. economic efficiency
- Weak efficiency vs. strong efficiency
- Local non-satiation
- 2003 Homework 1 - Question 2

Lecture 3 (4/11/05) - 3 pages

Key points

- Outline of the proofs of the first and second welfare theorems
- Geometric conditions for efficiency

  Proposition 8 *If* $\left[(x_i^*), (y_j^*), p\right]$ *is a PTE for* ε*, then it is a PTE for* $\hat{\varepsilon}$*.*

- Market socialism
- Preview of quasilinear general equilibrium

Notation and definitions

- $\hat{B}_i(x_i) = \left\{b : b = \sum_{k=1}^{l+1} \alpha_k b_k,\ \alpha_k \geq 0,\ \sum_{k=1}^{l+1} \alpha_k = 1\right\}$
- $\hat{\varepsilon} = \left[(X_i, \succeq_i, \omega_i), \left(\hat{Y}_j\right), (\theta_{ij})\right]$ with $\hat{B}_i(x_i)$
- $\omega_i = \bar{\omega}_i + r_i$
- $\bar{\omega}_i \equiv$ inalienable portion of the endowment
- $r_i \equiv$ alienable portion of the endowment
- $\alpha_i \equiv$ fraction of total endowment allocated to person i.

Lecture 4 (4/13/05) - 3 pages

Key points

- Replica invariance

  Proposition 9 *If* p *is an equilibrium price for* ε*, it is an equilibrium price for* ε^2 *if* ε *is replica invariant*

- Introduction to quasilinear general equilibrium
- Description of a consumer

  Proposition 10 $A(z, m)$ *is always convex* $\iff$ v *is concave*

Notation and definitions

- $w_i^\alpha(p) = \alpha_i(p \cdot \omega + \sup pY)$ - wealth function under market socialism
- $\varepsilon_i(p, w_i^\alpha(p))$ - demand correspondence under market socialism
- $U(\omega + z, m) = u(\omega + z) + m = v(z) + m$ - quasilinear preferences
- $v(z) = \begin{cases} u(\omega + z) & z \geq \omega \\ -\infty & else \end{cases}$
- $p \cdot z + m = 0$ - QL budget constraint with no outside gifts
- $A(z, m) = \{(z', m') : v(z') + m' \geq v(z) + m\}$ - weak preference set

Discussion 2 (4/13/15) - 5 pages

Key points

- **Proposition 11** *$\exists PTE \Rightarrow$ Replica invariance*
- 2003 Midterm - Question 1
- Quasilinear preferences
- 2003 Fall Comp - Question 5

Lecture 5 (4/18/05) - 4 pages

Key points

- Demand correspondence

  Proposition 12 *WLOG, we can assume that $w = 0$.*

- Subdifferential
- Concavified utility function

  Proposition 13 *If $p \in \partial v(z)$, then $v(z) = \hat{v}(z)$*

  Proposition 14 *Suppose v is concave and $\partial v(z) = \{p\}$. Then*

$$\nabla v(z) = \left(\frac{\partial v(z)}{\partial z_1}, ..., \frac{\partial v(z)}{\partial z_l}\right) = (p_1, ..., p_l) = p$$

- Indirect utility

  Proposition 15 $v(z) - pz = v^*(p) \iff p \in \partial v(z)$

- Convexity of indirect utility function
- Pareto optimality as the maximized sum of utility

Notation and definitions

- $d(v, p, w) = \{(z, m) : p \cdot z + m = w,\ v(z) + m \geq v(z') + m'\ \forall (z', m') \text{ s.t. } p \cdot z' + m' = w\}$ - demand correspondence
- $d(v, p, 0) = \{z : v(z) - p \cdot z \geq v(z') - p \cdot z'\ \forall z'\}$ - demand correspondence with no outside gifts
- $\partial v(z) = \{p : v(z) - p \cdot z \geq v(z') - p \cdot z'\ \forall z'\}$ - subdifferential of v at z
- $\hat{v}(z) = \sup\{\sum_k \lambda_k v(z_k) : \sum_k \lambda_k z_k = z,\ \lambda_k \geq 0,\ \sum_k \lambda_k = 1\}$ - concavified utility function
- $v^*(p) = \sup_z \{v(z) - p \cdot z\}$ - indirect utility function
- $\mathbb{V} = (v_1, ..., v_n)$ - completely characterizes a QL economy

  Definition 16 *A price taking equilibrium (for the economy $\mathbb{V}$) is a vector of allocations and prices $[(z_i, m_i), (p, 1)]$ s.t.*

  1. $(z_i, m_i) \in d(v_i, p, 0)\ \forall i,\ p \in \partial v_i(z_i)\ \forall i,$ *or* $v_i(z_i) - p \cdot z_i = v_i^*(p)\ \forall i$
  2. $\sum_{i=1}^n (z_i, m_i) = (0, 0) \in \mathbb{R}^{l+1}$

Lecture 6 (4/20/05) - 3 pages

Key points

- Commodity vs. individual as a margin of analysis

  Proposition 17 *Fix* (z_i) *s.t.* $\sum_i z_i = 0$. *Then* $\forall p$, $\sum_i v_i(z_i) \leq \sum_i v_i^*(p)$.

  Proposition 18 *If* $\sum_i v_i^*(p) = \sum_i v_i(z_i)$, *then* $\sum_i v_i(z_i) = v_I(0)$. *That is,* (z_i) *is PO.*

  Proposition 19 *A price taking equilibrium is Pareto optimal*

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