Econ142: Probabilistic Microeconomics

Problem Set 4

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1 Question 1 - Comparative Statics in the Insurance Market

The utility function of the decision maker is given by $u(x) = \ln x$. His wealth is w, and he wants to insure (maybe parts) of an asset (whose value is L) against a potential loss. The probability of loss is p, and the price of dollar insurance is q. Recall that $(\ln x)' = \frac{1}{x}$.

1.1 Part (a)

Find the optimal amount of insurance X the decision maker will buy

1.1.1 Answer

When the decision maker purchases X units of insurance, he faces the lottery

$$\begin{aligned} L(X) &= (w - qX, 1-p; w - qX - L + X, p) \\ &= (w - qX, 1-p; w + (1-q)X - L, p) \end{aligned}$$

First, we must note that, in general, $0 \leq X \leq L$. The first part of that inequality $(0 \leq X)$ implies that decision makers cannot SELL insurance. They can only buy it. The second inequality $(X \leq L)$ implies that there are no opportunities for overinsuring. Therefore, the consumer's problem is

$$\max_X Eu[L(X)] \text{ subject to } 0 \leq X \leq L$$

Ignoring, for the moment, the constraint (we will check the constraints later), we have:

$$\max_X (1-p)\ln(w - qX) + p\ln(w + (1-q)X - L)$$

Taking first order condition,

$$\frac{\partial Eu[L(X)]}{\partial X} = 0 \Rightarrow \frac{(1-p)}{w - qX}(-q) + \frac{p}{w + (1-q)X - L}(1-q) = 0$$

Rearranging and solving for X,

$$\begin{aligned} \frac{p(1-q)}{w + (1-q)X - L} &= \frac{(1-p)q}{w - qX} \qquad (1) \\ p(1-q)w - qp(1-p)X &= (1-p)qw + (1-p)q(1-q)X - (1-p)qL \\ (1-p)q(1-q)X + qp(1-q)X &= p(1-q)w - (1-p)qw + q(1-p)L \\ X[(1-p)q(1-q) + qp(1-q)] &= (p - pq)w - (qw - pqw) + q(1-p)L \\ X\left[(q - qp)(1-q) + qp - q^2p\right] &= pw - pqw - qw + pqw + q(1-p)L \\ X\left[q - q^2 - qp + q^2p + qp - q^2p\right] &= pw - qw + q(1-p)L \\ X\left[q - q^2\right] &= w(p-q) + q(1-p)L \\ X &= \frac{w(p-q) + q(1-p)L}{q(1-q)} \\ &= \frac{(1-p)}{(1-q)}L - \frac{(q-p)}{q(1-q)}w \end{aligned}$$

1.2 Part (b)

Verify that $X = L$ iff $q \leq p$.

1.2.1 Answer

Suppose $q = p$. Then

$$\begin{aligned} X &= \frac{(1-p)}{(1-q)}L - \frac{(q-p)}{q(1-q)}w \\ &= \frac{(1-q)}{(1-q)}L - \frac{(q-q)}{q(1-q)}w \\ &= L - 0 \\ &= L \end{aligned}$$

It is obvious, but somewhat difficult to prove that if $q < p$, then $X > L$. But recall that $X \leq L$ since there cannot be any overinsurance. Therefore, $X = L$.

Suppose $X = L$. Using (1),

$$\begin{aligned} \frac{p(1-q)}{w + (1-q)X - L} &= \frac{(1-p)q}{w - qX} \\ \frac{p(1-q)}{w + (1-q)L - L} &= \frac{(1-p)q}{w - qL} \\ \frac{p(1-q)}{w - qL} &= \frac{(1-p)q}{w - qL} \\ p - pq &= q - qp \\ p &= q \end{aligned}$$

1.3 Part (c)

Assuming that $q > p$, what happens to the optimal X when w increases?

1.3.1 Answer

From part (a), we know

$$X = \frac{(1-p)}{(1-q)}L - \frac{(q-p)}{q(1-q)}w$$

Therefore,

$$\frac{\partial X}{\partial w} = -\frac{q-p}{q(1-q)} < 0$$

Since $q - p > 0$, $q \geq 0$, and $1 - q \geq 0$. This implies that insurance is an inferior good.

1.4 Part (d)

What happens to the optimal X when p increases?

1.4.1 Answer

From part (a), we know

$$X = \frac{(1-p)}{(1-q)}L - \frac{(q-p)}{q(1-q)}w$$

Therefore,

$$\begin{aligned} \frac{\partial X}{\partial p} &= -\frac{L}{1-q} + \frac{w}{q(1-q)} \\ &= \frac{1}{1-q}[w - qL] \end{aligned}$$

If we assume $w > qL$, that is, the wealth level is greater than the cost of full insurance, which is a very reasonable assumption, then, since $1 - q > 0$,

$$\frac{\partial X}{\partial p} > 0$$

That is, as the probability of the bad event occurring increases, the decision maker will want to purchase more insurance.

1.5 Part (e)

What happens to the optimal X when L increases?

1.5.1 Answer

From part (a), we know

$$X = \frac{(1-p)}{(1-q)}L - \frac{(q-p)}{q(1-q)}w$$

Therefore,

$$\frac{\partial X}{\partial L} = \frac{1-p}{1-q} > 0$$

Since $1 - p \geq 0$ and $1 - q \geq 0$. That is, as the losses associated with the bad event increase, the decision maker will purchase more insurance.

1.6 Part (f)

Why didn't I ask you what happens to the optimal X when w increases?

1.6.1 Answer

You did ask what would happen to the optimal X when w increases. If we reinterpret this question as "Why didn't I ask you what happens to the optimal X when q increases?" the response would be that there are two different counteracting effects, just like in any consumer optimization problem. The substitution effect of an increase in q will lead to a decrease in the quantity of X purchased. From part (c), we see that the income effect would be in the opposite direction. That is, an increase in the price of insurance would lead to a decrease in wealth in some sense. This would in turn lead to an increase in the quantity of X purchased. The result of these counteracting effects would depend on the relative values of the parameters.