

## Econ106P: Pricing and Strategy

### Problem Set 3

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## 1 Question 1

A power company sells electricity in three periods that have the following demand functions:

$$P_1 = 44 - Q_1$$

$$P_2 = 21 - Q_2$$

$$P_3 = 15 - Q_3$$

The marginal cost of providing power in each period is

$$MC_1 = 2, MC_2 = 1, MC_3 = 1$$

The marginal cost of a unit of capacity is  $MC_J = Q$  ( $TC_J = 0.5Q^2$ ). A unit of capacity can provide one unit of output for each period.

### 1.1 Part (a)

Calculate NMV for each period. Assuming that the firm sells equal amounts of power in each period, what is the quantity that maximizes the social total? What is NMV in each period at this quantity? What do these values indicate to you?

#### 1.1.1 Answer

Recall that  $NMV_i = MV_i - MC_i$  and  $MV_i = P_i$ . Therefore, we have:

$$NMV_1 = 44 - Q_1 - 2$$

$$= 42 - Q_1$$

$$NMV_2 = 21 - Q_2 - 1$$

$$= 20 - Q_2$$

$$NMV_3 = 15 - Q_3 - 1$$

$$= 14 - Q_3$$

Assuming  $Q_1 = Q_2 = Q_3 \equiv Q$ , if we vertically sum the  $NMV_i$ , we get:

$$NMV_1 + NMV_2 + NMV_3 = MC_J$$

$$(42 - Q) + (20 - Q) + (14 - Q) = Q$$

$$76 - 3Q = Q$$

$$4Q = 76$$

$$Q = 19$$

Evaluating the  $NMV_i$  at  $Q$ ,

$$NMV_1 = 42 - 19 = 23$$

$$NMV_2 = 20 - 19 = 1$$

$$NMV_3 = 14 - 19 = -5 < 0$$

Since  $NMV_3 < 0$ , it follows that we are overproducing in period 3. Also, we know that  $NMV_1 + NMV_2 = 24 > 19 = Q = MC_J$ , so we are underproducing these two goods. Note that we do not yet know whether we are producing too little of good 2 yet, just that we are producing too little of the sum of these two goods.

## 1.2 Part (b)

Relaxing the assumption that quantities be equal in each period, find the quantities that maximize the social total. Find prices for each period. What does this answer tell you about how the joint capacity cost should be allocated?

### 1.2.1 Answer

Since  $NMV_3 < 0$ , we must set  $NMV_3 = 0$  to solve for  $Q_3$ .

$$\begin{aligned}NMV_3 &= 14 - Q_3 = 0 \\Q_3 &= 14\end{aligned}$$

Suppose now that  $Q_1 = Q_2 = Q$ .

$$\begin{aligned}NMV_1 + NMV_2 &= MC_J \\(42 - Q) + (20 - Q) &= Q \\62 - 2Q &= Q \\3Q &= 62 \\Q &= 20\frac{2}{3}\end{aligned}$$

Evaluating  $NMV_1$  and  $NMV_2$  at  $Q = 20\frac{2}{3}$ ,

$$\begin{aligned}NMV_1 &= 42 - 20\frac{2}{3} = 21\frac{1}{3} \\NMV_2 &= 20 - 20\frac{2}{3} = -\frac{2}{3} < 0\end{aligned}$$

Since  $NMV_2 < 0$ , we are overproducing during period 2 and since  $NMV_1 = 21\frac{1}{3} > 20\frac{2}{3} = MC_J$ , we are underproducing during period 1. Thus, we should set  $NMV_2 = 0$  to solve for  $Q_2$  and set  $NMV_1 = MC_J$ , allocating the entire joint cost to period 1, since this is the period during which the capacity constraint is binding, to find  $Q_1$ .

$$\begin{aligned}NMV_2 &= 20 - Q_2 = 0 \\Q_2 &= 20 \\NMV_1 &= 42 - Q_1 = Q_1 \\2Q_1 &= 42 \\Q_1 &= 21\end{aligned}$$

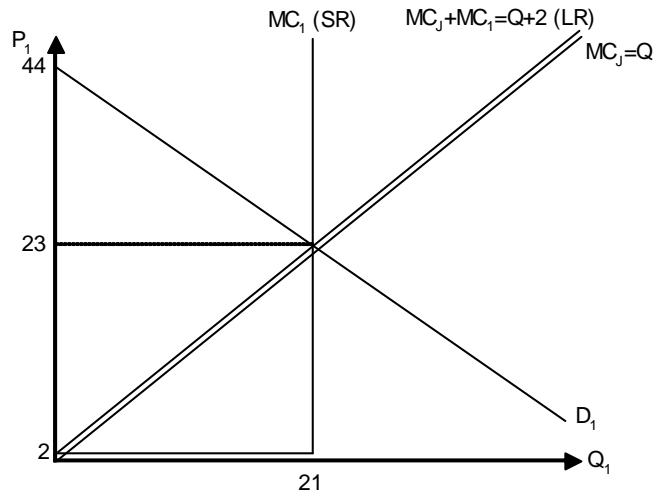
Solving for prices, we have:

$$\begin{aligned}P_1 &= 44 - Q_1 = 44 - 21 = 23 \\P_2 &= 21 - Q_2 = 21 - 20 = 1 \\P_3 &= 15 - Q_3 = 15 - 14 = 1\end{aligned}$$

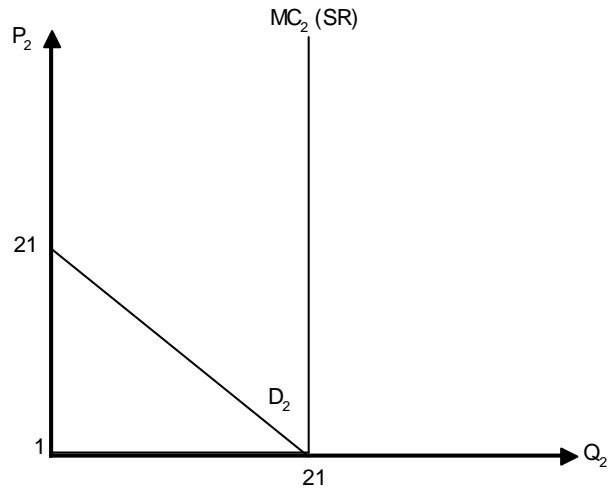
## 1.3 Part (c)

Assume that the ST-maximizing level of capacity (from part b) has been installed and that therefore the capacity costs are now sunk. Taking into account the capacity constraint in the short run, what does the marginal cost curve ( $MC_1, MC_2, MC_3$ ) look like for each period? Use separate graphs for each period. Now draw in the demand curve for each period along with the  $MC_J$  curve for the period one graph. What does the long-run marginal cost curve look like for period one?

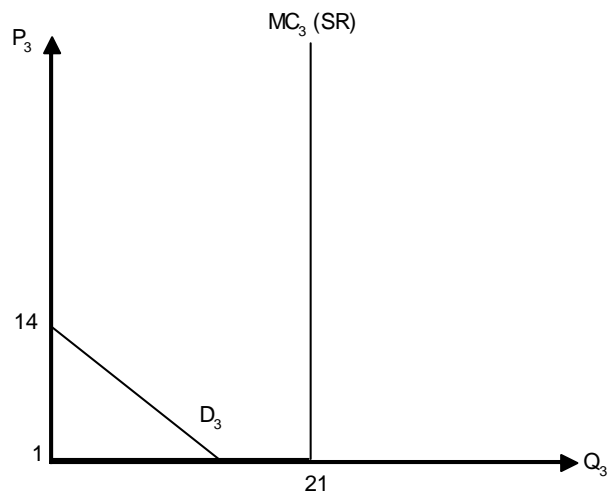
1.3.1 Answer



Period 1 with joint costs



Period 2



Period 3

### 1.4 Part (d)

Suppose the demand in period one increased to  $P_1 = 50 - Q_1$ . Using your graph as a guide, find the new ST-maximizing quantity and price in the short-run and the long-run in period 1. Will this increase in demand for period one affect the price charged in periods 2 or 3? Explain.

#### 1.4.1 Answer

In the short-run, we will have

$$\begin{aligned} P_1 &= 50 - Q_1 \\ &= 50 - 21 \\ &= 29 \end{aligned}$$

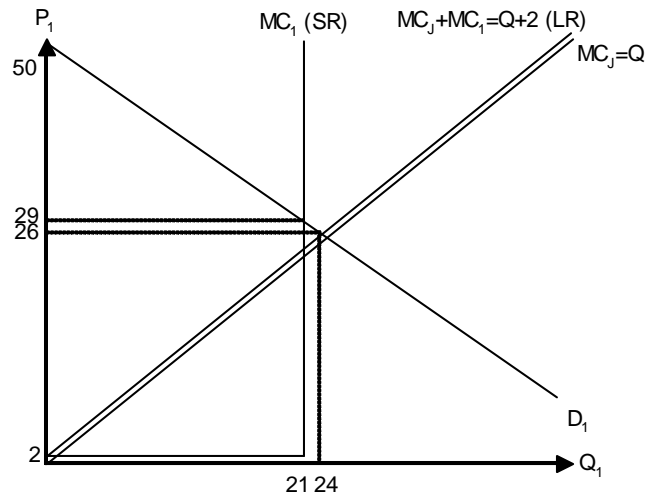
In the long-run, the new price and quantity can be found by equating demand and the long-run supply.

$$\begin{aligned} P_1 &= MC_J + MC_1 \\ 50 - Q_1 &= Q_1 + 2 \\ 2Q_1 &= 48 \\ Q_1 &= 24 \end{aligned}$$

At this quantity, the price will be:

$$\begin{aligned} P_1 &= 50 - Q_1 \\ &= 50 - 24 \\ &= 26 \end{aligned}$$

Graphically, we have:



Period 1 with increased demand

Since the only change brought about by this increase in demand that is relevant for periods 2 and 3 is that the capacity will be increased from 21 to 24, there will be no changes in those periods, since the capacity constraint was not binding to begin with.

### 1.5 Part (e)

Suppose this firm can purchase power in the wholesale market at a price of 19 in period 1. How will this opportunity affect the quantity of capacity, quantity of power sold, and the price in period one in the long run? Again assume that  $P_1 = 44 - Q_1$ . Show your answers on a graph.

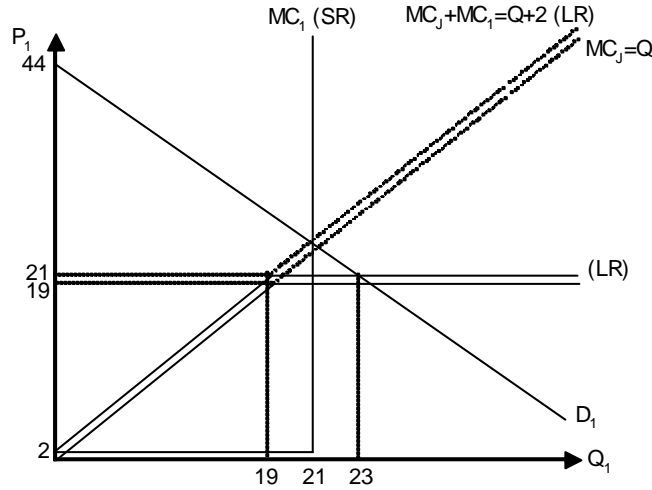
**1.5.1 Answer**

The long-run supply curve is constructed by taking the lower envelope of the  $MC_J + MC_1$  curve and the  $MC_1 + 19$  curve. This curve intersects the demand curve beyond the kink (where  $P_1 = 21$ )

$$\begin{aligned} P_1 &= 44 - Q_1 = 21 \\ Q_1 &= 23 \end{aligned}$$

Which gives us the quantity of power supplied in period 1 of  $Q_1 = 23$ .

At the long run price of  $P_1 = 21$ , we have that  $Q_{capacity} = 21 - 2 = 19$ . It then follows that the firm would be purchasing  $Q_1 - Q_{capacity} = 23 - 19 = 4$  units from the outside market.



Period 1 with outside market for capacity