Econ106P: Pricing and Strategy

Problem Set 2

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1 Question 1

The Welsh Sword Company specializes in the production of replica swords for historical battle reenactors. It has four productoin plants in Cardiff, Swansea, Barmouth, and Aberystwyth.

The swords are all shipped to one of five distribution centers in London, New York, Boston, Paris, and Rome. The transportation costs (in dollars) between plants and distribution centers are as follows:

	London	New York	Boston	Paris	Rome
Cardiff	49.2	112.3	110.5	60.5	86.7
Swansea	51.9	110.1	106.6	63.7	88.2
Barmouth	52.3	111.8	115.2	62.4	92.5
Aberystwyth	53.9	113.9	118.8	64.3	91.6

The maximum capacity of the Cardiff plant is 2145; the capacity of the Swansea plant is 1921; the capacity of the Barmouth plant is 1838; and the capacity fo the Aberystwyth plant is 2505. The minimum required shipments to London, New York, Boston, Paris, and Rome are 1572, 1342, 1506, 1688, 1822 respectively.

1.1 Part (a)

The company's objective is to minimize the cost of transporting its product from its plants to its distribution center while satisfying the above constraints. Write out the objective function and the constraints.

1.1.1 Answer

Define:

$$\begin{aligned}
C &\equiv Cardiff \\
S &\equiv Swansea \\
B &\equiv Barmouth \\
A &\equiv Aberystwyth \\
L &\equiv London \\
N &\equiv New\ York \\
B' &\equiv Boston \\
P &\equiv Paris \\
R &\equiv Rome \\
(i \to j) &\equiv \text{Quantity transported from } i \text{ to } j \text{ where} \\
i &\in \{C, S, B, A\} \text{ and } j \in \{L, N, B', P, R\}.
\end{aligned}$$

Objective function:

$$\begin{aligned}
&\min 49.2\,(C \to L) + 51.9\,(S \to L) + 52.3\,(B \to L) + 53.9\,(A \to L)\\
&+112.3\,(C \to N) + 110.1\,(S \to N) + 111.8\,(B \to N) + 113.9\,(A \to N)\\
&+110.5\,(C \to B') + 106.6\,(S \to B') + 115.2\,(B \to B') + 118.8\,(A \to B')\\
&+60.5\,(C \to P) + 63.7\,(S \to P) + 62.4\,(B \to P) + 64.3\,(A \to P)\\
&+86.7\,(C \to R) + 88.2\,(S \to R) + 92.5\,(B \to R) + 91.6\,(A \to R)
\end{aligned}$$

Constraints:
Capacity Constraints:

$$\begin{aligned}
\text{Cardiff Capacity} &: (C \to L) + (C \to N) + (C \to B') + (C \to P) + (C \to R) \leq 2145\\
\text{Swansea Capacity} &: (S \to L) + (S \to N) + (S \to B') + (S \to P) + (S \to R) \leq 1921\\
\text{Barmouth Capacity} &: (B \to L) + (B \to N) + (B \to B') + (B \to P) + (B \to R) \leq 1838\\
\text{Aberystwyth Capacity} &: (A \to L) + (A \to N) + (A \to B') + (A \to P) + (A \to R) \leq 2505
\end{aligned}$$

Transportation Requirement Constraints:

$$\begin{aligned}
\text{London Requirement} &: (C \to L) + (S \to L) + (B \to L) + (A \to L) \geq 1572\\
\text{New York Requirement} &: (C \to N) + (S \to N) + (B \to N) + (A \to N) \geq 1342\\
\text{Boston Requirement} &: (C \to B') + (S \to B') + (B \to B') + (A \to B') \geq 1506\\
\text{Paris Requirement} &: (C \to P) + (S \to P) + (B \to P) + (A \to P) \geq 1688\\
\text{Rome Requirement} &: (C \to R) + (S \to R) + (B \to R) + (A \to R) \geq 1822
\end{aligned}$$

Non-negativity Constraints:

$$\begin{aligned}
(C \to L), (S \to L), (B \to L), (A \to L) &\geq 0\\
(C \to N), (S \to N), (B \to N), (A \to N) &\geq 0\\
(C \to B'), (S \to B'), (B \to B'), (A \to B') &\geq 0\\
(C \to P), (S \to P), (B \to P), (A \to P) &\geq 0\\
(C \to R), (S \to R), (B \to R), (A \to R) &\geq 0
\end{aligned}$$

1.2 Part (b)

Find the cost-minimizing solution using EXCEL's Solver. Hand in copies of the answer report and the sensitivity report.

1.2.1 Answer

Using EXCEL's Solver, we have that the minimum transportation cost is

657857.3

Which occurs when

	London	New York	Boston	Paris	Rome
Cardiff	323	0	0	0	1822
Swansea	0	415	1506	0	0
Barmouth	0	927	0	911	0
Aberystwyth	1249	0	0	777	0

The answer report and the sensitivity reports are attached.

1.3 Part (c)

How do you interpret the shadow prices for the capacity constraints. Would it be profitable to add another unit of capacity to the Cardiff plant if the cost of an additional unit of capacity is \$4? Explain your answer with reference to the sensitivity report.

1.3.1 Answer

Formally, the interpretation of a shadow price for a constraint is

$$\frac{\Delta\text{objective function}}{\Delta\text{constraint}}$$

In the case of the shadow prices for the capacity constraints, the interpretation is the savings in total cost if we were to increase the capacity constraint by one unit. According to the sensitivity report, the shadow price of the Cardiff capacity constraint is -4.7. That is, if we increase the capacity of the Cardiff plant by one unit, we can save \$4.70. For a cost of \$4, it is profitable to add another unit of capacity to the Cardiff plant.

1.4 Part (d)

Explain the value of Swansea's shadow price with reference to the changing pattern of shipments if Swansea had one more unit of capacity.

1.4.1 Answer

If we increase the capacity of the Swansea plant by one unit, the following changes occur:

$$\begin{aligned}
(S \to N) &: \quad 415 \to 416 \quad (+1) \text{ resulting in } +110.1 \text{ costs} \\
(B \to N) &: \quad 927 \to 926 \quad (-1) \text{ resulting in } -111.8 \text{ costs} \\
(B \to P) &: \quad 911 \to 912 \quad (+1) \text{ resulting in } +62.4 \text{ costs} \\
(A \to P) &: \quad 777 \to 776 \quad (-1) \text{ resulting in } -64.3 \text{ costs}
\end{aligned}$$

The sum of these effects is -3.6. That is, increasing the capacity of the Swansea plant by one unit leads to a decrease in total costs by 3.6. Not coincidentally, the shadow price of the Swansea capacity constraint is -3.6.

1.5 Part (e)

Explain the value of Boston's shadow price with reference to the changing pattern of shipments if the minimum shipping requirement to Boston increased by one unit.

1.5.1 Answer

If we increase the transportation requirement for Boston by one unit, the following changes occur:

$$\begin{aligned}
(S \to N) &: \quad 415 \to 414 \quad (-1) \text{ resulting in } +110.1 \text{ costs} \\
(S \to B) &: \quad 1506 \to 1507 \quad (+1) \text{ resulting in } +106.6 \text{ costs} \\
(B \to N) &: \quad 927 \to 928 \quad (+1) \text{ resulting in } +111.8 \text{ costs} \\
(B \to P) &: \quad 911 \to 910 \quad (-1) \text{ resulting in } -62.4 \text{ costs} \\
(A \to P) &: \quad 777 \to 778 \quad (+1) \text{ resulting in } +64.3 \text{ costs}
\end{aligned}$$

The sum of these effects is $+110.2$. That is, increasing the transportation requirement for Boston by one unit leads to an increase in total costs of 110.2. Not coincidentally, the shadow price of the Boston requirement constraint is $+110.2$.

1.6 Part (f)

By how much could the cost of shipping from Swansea to Boston change by without changing your initial answer? Explain your answer with reference to the sensitivity report.

1.6.1 Answer

If, by "without changing your initial answer," it is meant "without changing any of the $(i \to j)$s," then from the sensitivity report, we see that the allowable increase of the cost of shipping from Swansea to Boston is 5 units. That is, we can increase the cost of shipping from Swansea to Boston by 5 units without changing any of the $(i \to j)$s.

2 Question 2

Assume that a portfolio manager's goal is to construct a portfolio of securities that offers the highest expected after-tax return, subject to the following requirements: a. The portfolio's average quality rating is at least 3.2 and b. the portfolio's average maturity is at least 1.5 years but no more than 2.9 years. The relevant information for each security is given below

Bond Category	Quality Rating	Maturity (Years)	Yield (Percent)
Treasury Bills	5	0.5	2.4
Treasury Bonds	4.7	4.5	4.1
Corporate Bonds	3.6	3.8	6.82
Municipal Bonds	3	2.5	5.25
Junk Bonds	0.7	3	9.9

Let the choice variables be defined as B = proportion of Treasury Bills in the portfolio, T = proportion of Treasury Bonds in the portfolio, C = proportion of Corporate Bonds in the portfolio, M = proportion of Municipal Bonds in the portfolio, J = proportion of Junk Bonds in the portfolio. (Note: One of your constraints will be $B + T + C + M + J = 1$)

2.1 Part (a)

Write out the appropriate objective function and constraints.

2.1.1 Answer

Objective function:

$$\max 2.4B + 4.1T + 6.82C + 5.25M + 9.9J$$

Constraints:

$$\begin{aligned}
\text{Quantity} &: \quad B + T + C + M + J = 1 \\
\text{Quality} &: \quad 5B + 4.7T + 3.6C + 3M + 0.7J \geq 3.2 \\
\text{Maturity (Lower)} &: \quad 0.5B + 4.5T + 3.8C + 2.5M + 3J \geq 1.5 \\
\text{Maturity (Upper)} &: \quad 0.5B + 4.5T + 3.8C + 2.5M + 3J \leq 2.9 \\
\text{Non-negativity} &: \quad B, T, C, M, J \geq 0
\end{aligned}$$

2.2 Part (b)

Find the company's most profitable production plan using EXCEL's Solver. Hand in the copies of the Sensitivity Report and the Answer Report.

2.2.1 Answer

Using EXCEL's Solver, we have that the maximum average yield is

$$6.62$$

Which occurs when

$$\begin{aligned} B &= 0.21 \\ T &= 0 \\ C &= 0.54 \\ M &= 0 \\ J &= 0.24 \end{aligned}$$

Note: Numbers may not add up to 1 due to rounding errors.

2.3 Part (c)

Which (if any) of the constraints are non-binding?

2.3.1 Answer

According to the answer report, the Maturity (Lower) constraint and the non-negativity constraints for B, C, and M are non-binding

2.4 Part (d)

How do you interpret the shadow price for the quality rating constraint?

2.4.1 Answer

Formally, the interpretation of a shadow price for a constraint is

$$\frac{\Delta \text{objective function}}{\Delta \text{constraint}}$$

In the case of the shadow price for the quality rating constraint, the interpretation is the drop in average yield resulting in a unit increase in the quality rating requirement. According to the sensitivity report, a unit increase in the Quality constraint results in a decrease in the average yield by 1.28. On a slightly technical note, the allowable increase for this constraint is only 0.78, which means that a unit increase in the Quality constraint is not possible without changing the shadow price. Nevertheless, a 0.78 unit increase will result in a $(0.78)(1.28)$ unit decrease in the average yield.

2.5 Part (e)

Use the information in the Sensitivity Report to answer the following questions. If the quality rating had to be at least 3.3, what would be the predicted change in the portfolio rate of return?

If the upper limit on the average maturity constraint was changed to 3.2, what would be the predicted change in the portfolio rate of return?

Would the proportion of each security held change if the yield on Treasury Bills changed to 3.5%?

If the quality rating of Corporate Bonds increased to 4, would C increase? How do you explain this result?

2.5.1 Answer

If the quality rating had to be at least 3.3, the change in the portfolio rate of return would be

$$(-1.28)(3.3 - 3.2) = -0.128$$

If the upper limit on the average maturity constraint was changed to 3.2, the change in the portfolio rate of return would be

$$(0.8)(3.2 - 2.9) = +0.24$$

Since the allowable increase of the yield on Treasury Bills is 2.93 (its current value being 2.4), it follows that if the yield on Treasury Bills increased to 3.5%, it would not change the optimal proportions of the security holdings.

Actually, if the quality rating of Corporate Bonds increased to 4, C would decrease (from 0.54 to 0.50). This would result because the increase in the quality rating of Corporate Bonds would increase the average quality rating of the portfolio, allowing an increase in the holdings of Junk Bonds (which have a much higher yield) until the constraint once again becomes binding. The final portfolio would be:

$$\begin{aligned} B &= 0.20 \\ T &= 0 \\ C &= 0.50 \\ M &= 0 \\ J &= 0.3 \end{aligned}$$

And the average yield would be 6.87.