Econ106P: Pricing and Strategy

Problem Set 1

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1 Question 1

Consider CIPEC (copper cartel) as the dominant firm in an industry consisting of itself and many other small firms. The market demand is given by $Q_M = 1600 - P$. The supply curve for the fringe firms is $Q_{FF} = -400 + P$ and the MC of the dominant firm $MC = 200 + Q_{Dom}$.

1.1 Part (a)

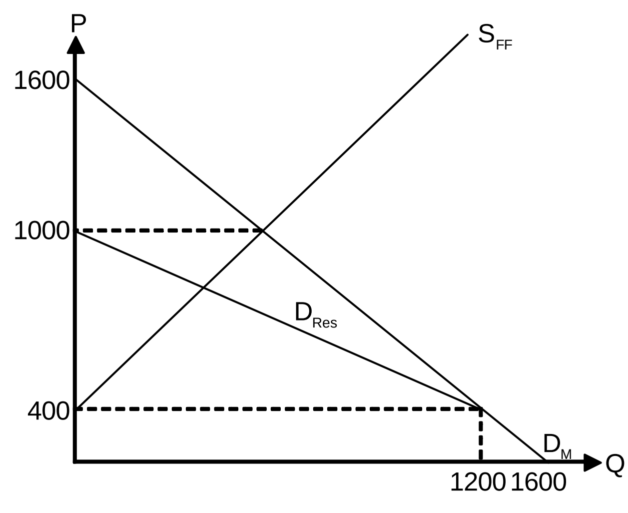
Find the dominant firm's residual demand curve.

1.1.1 Answer

Mathematically,

$$\begin{aligned} Q_{RES} &= Q_M - \max\{Q_{FF}, 0\} \\ &= \begin{cases} 1600 - P - (-400 + P) & 0 \le Q \le 1200 \\ 1600 - P & 1200 \le Q \le 1600 \end{cases} \\ &= \begin{cases} 2000 - 2P & 0 \le Q \le 1200 \\ 1600 - P & 1200 \le Q \le 1600 \end{cases} \end{aligned}$$

Graphically, we have:



1.2 Part (b)

Find price, dominant firm output, and fringe firm output in this industry.

1.2.1 Answer

In the relevant region, we have for the dominant firm (from (a))

$$\begin{aligned} 2P &= 2000 - Q_{DOM}. \text{ Solving for } P \text{, we have,} \\ P &= 1000 - \frac{1}{2}Q_{DOM} \text{ and thus,} \\ MR_{DOM} &= 1000 - Q_{DOM} \end{aligned}$$

Setting $MC_{DOM} = MR_{DOM}$,

$$\begin{aligned} 1000 - Q_{DOM} &= 200 + Q_{DOM} \\ 800 &= 2Q_{DOM} \\ Q_{DOM} &= 400 \end{aligned}$$

Solving for prices, we have

$$\begin{aligned} P &= 1000 - \frac{1}{2}Q_{DOM} \\ &= 1000 - 200 \\ &= 800 \end{aligned}$$

At these prices, the fringe firms will supply

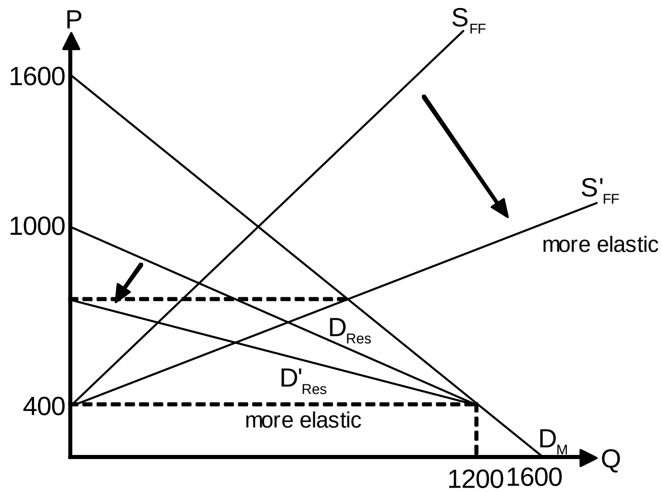
$$\begin{aligned} Q_{FF} &= -400 + P \\ &= -400 + 800 \\ &= 400 \end{aligned}$$

1.3 Part (c)

As the fringe supply curve becomes more elastic, what happens to the elasticity of the residual demand curve? Justify your answer with the aid of a graph

1.3.1 Answer

When the fringe supply curve becomes more elastic, the residual demand curve becomes more elastic. I have illustrated a change in elasticity of the fringe firm supply curve below, holding constant its vertical intercept:



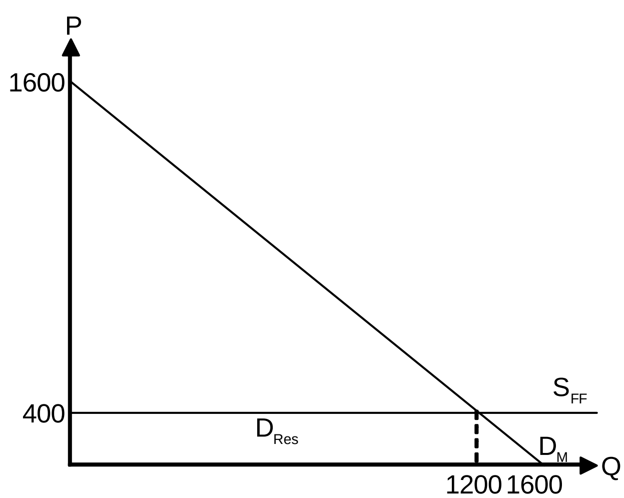
1.4 Part (d)

What happens to the residual demand curve if the fringe supply curve becomes perfectly elastic (horizontal)? Justify your answer with a graph and assume that the fringe supply curve is perfectly elastic at \$400. Find dominant firm output, price, and fringe firm output.

1.4.1 Answer

Verbally, if the fringe supply curve becomes perfectly elastic at \$400, then at all levels of output below 1200, the most the dominant firm could charge is \$400. Therefore, the residual demand curve would coincide with the fringe supply curve until the "kink" at the quantity $Q = 1200$ after which, it would coincide with the market demand curve.

The graph of this is:



Since the residual demand curve is flat, the marginal revenue for the dominant firm when $Q \leq 1200$ is also flat. Therefore, we have:

$$\begin{aligned} MC &= MR \\ 200 + Q_{DOM} &= 400 \\ Q_{DOM} &= 200 \end{aligned}$$

The equilibrium price would be 400 and therefore

$$\begin{aligned} Q_M &= 1600 - 400 \\ &= 1200 \end{aligned}$$

Since we have that $Q_M = Q_{DOM} + Q_{FF}$, it follows that the fringe firms will supply $Q_{FF} = 800$.

1.5 Part (e)

How would your answer to part (d) change if the dominant firm's marginal cost function was $MC = 200 + \frac{1}{6}Q_{DOM}$?

1.5.1 Answer

The derivation of the residual demand curve remains the same. Marginal revenue would still be constant at 400 in the relevant region, so we would have:

$$\begin{aligned} MC &= MR \\ 200 + \frac{1}{6}Q_{DOM} &= 400 \\ \frac{1}{6}Q_{DOM} &= 200 \\ Q_{DOM} &= 1200 \end{aligned}$$

The prices will remain constant at 400 and thus the market demand will still be $Q_M = 1200$. Since $Q_M = Q_{DOM} + Q_{FF}$, it follows that $Q_{FF} = 0$.

1.6 Part (f)

Let us continue to assume that the market demand curve is given by $Q_M = 1600 - P$ and the fringe supply curve is $Q_{FF} = -400 + P$. However, now we shall assume that there are TWO large firms with market power who Cournot compete. Each firm has a constant MC of 200. Find output for these two firms, price, and fringe firm output.

1.6.1 Answer

The analysis begins the same as in part (a). The residual demand curve for the Cournot duopolists will be the horizontal difference between market demand and fringe supply.

$$\begin{aligned} Q_{RES} &= 2000 - 2P \\ P &= 1000 - \frac{1}{2}Q_{RES} \end{aligned}$$

Using the residual demand curve, the Cournot analysis follows.

$$\begin{aligned} \pi_1 &= (P - 200)(Q_1) \\ &= \left(1000 - \frac{1}{2}(Q_1 + Q_2) - 200\right) Q_1 \\ &= 800Q_1 - \frac{1}{2}Q_1^2 - \frac{1}{2}Q_1Q_2 \end{aligned}$$

Taking first order conditions, we have:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial Q_1} &= 800 - Q_1 - \frac{1}{2}Q_2 = 0 \\
Q_1 &= 800 - \frac{1}{2}Q_2 \text{ Firm 1's reaction function} \\
Q_2 &= 800 - \frac{1}{2}Q_1 \text{ Firm 2's reaction function by symmetry}
\end{aligned}$$

Substituting Firm 2's reaction function into Firm 1's, we have:

$$\begin{aligned}
Q_1 &= 800 - \frac{1}{2}\left(800 - \frac{1}{2}Q_1\right) \\
&= 800 - 400 + \frac{1}{4}Q_1 \\
\frac{3}{4}Q_1 &= 400 \\
Q_1 &= \frac{1600}{3} \text{ and therefore} \\
Q_2 &= \frac{1600}{3} \text{ by symmetry}
\end{aligned}$$

The market price will then be

$$\begin{aligned}
P &= 1000 - \frac{1}{2}\left(Q_1 + Q_2\right) \\
&= 1000 - \frac{1}{2}\left(\frac{2}{3}1600\right) \\
&= 1000 - \frac{1600}{3} \\
&= 466\frac{2}{3}
\end{aligned}$$

At this price, the fringe firms will supply

$$\begin{aligned}
Q_{FF} &= -400 + P \\
&= -400 + 466\frac{2}{3} \\
&= 66\frac{2}{3}
\end{aligned}$$

2 Question 2

Assume a market demand of $P = 20 - Q$ and that there are two identical firms in the industry. Marginal cost is constant at \$2 and there are no fixed costs.

2.1 Part (a)

Find quantities, price and profit for each firm in the Cournot model.

2.1.1 Answer

Let market supply be $Q = Q_1 + Q_2$. Then we have

$$\begin{aligned}
\pi_1 &= (P - MC_1)\, Q_1 \\
&= (20 - Q - 2)\, Q_1 \\
&= (18 - Q_1 - Q_2)\, Q_1 \\
&= 18Q_1 - Q_1^2 - Q_1 Q_2
\end{aligned}$$

In order to derive the reaction curves, we take first order condition

$$\begin{aligned}
\frac{\partial \pi_1}{\partial Q_1} &= 18 - 2Q_1 - Q_2 = 0 \\
Q_1 &= 9 - \frac{1}{2}Q_2 \text{ Firm 1's reaction function} \\
Q_2 &= 9 - \frac{1}{2}Q_1 \text{ Firm 2's reaction function by symmetry}
\end{aligned}$$

Substituting Firm 2's reaction function into Firm 1's, we have:

$$\begin{aligned}
Q_1 &= 9 - \frac{1}{2}Q_2 \\
&= 9 - \frac{1}{2}\left(9 - \frac{1}{2}Q_1\right) \\
&= 9 - \frac{9}{2} + \frac{1}{4}Q_1 \\
\frac{3}{4}Q_1 &= \frac{9}{2} \\
Q_1 &= 6 \text{ and therefore} \\
Q_2 &= 6 \text{ by symmetry}
\end{aligned}$$

The prices are therefore

$$\begin{aligned}
P &= 20 - Q \\
&= 20 - Q_1 - Q_2 \\
&= 20 - 6 - 6 \\
&= 8
\end{aligned}$$

And thus the profits are

$$\begin{aligned}
\pi_1 &= (P - MC_1)\,(Q_1) \\
&= (8 - 2)\,(6) \\
&= 36 \text{ and} \\
\pi_2 &= 36 \text{ by symmetry}
\end{aligned}$$

2.2 Part (b)

How much would each firm produce if there were 17 identical firms? What is the price in this case?

2.2.1 Answer

The general formula for the market output in the n-producer Cournot case is

$$Q = Q_1 + ... + Q_n = \frac{n}{n+1} Q_{competitive}$$

The output that would occur under competitive conditions is:

$$\begin{aligned} P &= MC \\ 20 - Q &= 2 \\ Q_{competitive} &= 18 \end{aligned}$$

Therefore, if $n = 17$, the market output is:

$$\begin{aligned} Q &= \frac{17}{18} Q_{competitive} \\ &= \frac{17}{18} 18 \\ &= 17 \end{aligned}$$

Since each firm is identical, each firm would produce $\frac{1}{17}$ of the market output. That is, each firm would produce exactly 1 unit. ($Q_1 = ... = Q_{17} = 1$)

The market price would therefore be

$$\begin{aligned} P &= 20 - Q \\ &= 20 - 17 \\ &= 3 \end{aligned}$$

2.3 Part (c)

The Cournot model assumes what about $\frac{dQ_2}{dQ_1}$? Suppose we assume that $\frac{dQ_2}{dQ_1} = +1$ (and also $\frac{dQ_1}{dQ_2} = +1$) What is the outcome now and how does it compare with the monopoly (or cartel) outcome? Explain why a cartel is unstable under the Cournot assumption about $\frac{dQ_2}{dQ_1}$ (and under the assumption of a one-shot game) but is stable under the assumption $\frac{dQ_2}{dQ_1} = +1$. USE reaction curves for each case to justify your answer. What is the intuition behind this outcome?

2.3.1 Answer

The Cournot assumption is that each firm takes the other firm's output as a given. Mathematically, this says that $\frac{\partial Q_i}{\partial Q_j} = 0$ for $i \neq j$. If we instead assume that $\frac{\partial Q_i}{\partial Q_j} = 1$ for $i \neq j$, we will have different reaction functions.

$$\begin{aligned} \pi_1 &= (P - MC_1) Q_1 \\ &= (20 - Q_1 - Q_2 - 2) Q_1 \\ &= 18Q_1 - Q_1^2 - Q_1 Q_2 \end{aligned}$$

Taking first order conditions with $\frac{\partial Q_2}{\partial Q_j} = 1$, we have

$$\begin{aligned}
\frac{\partial \pi_1}{\partial Q_1} &= 18 - 2Q_1 - Q_1 - Q_2 = 0 \\
3Q_1 &= 18 - Q_2 \\
Q_1 &= 6 - \frac{1}{3}Q_2 \text{ Firm 1's reaction function} \\
Q_2 &= 6 - \frac{1}{3}Q_1 \text{ Firm 2's reaction function by symmetry}
\end{aligned}$$

Substituting firm 2's reaction function into firm 1's, we have:

$$\begin{aligned}
Q_1 &= 6 - \frac{1}{3}\left(6 - \frac{1}{3}Q_1\right) \\
&= 6 - 2 + \frac{1}{9}Q_1 \\
\frac{8}{9}Q_1 &= 4 \\
Q_1 &= \frac{9}{2} \text{ and therefore} \\
Q_2 &= \frac{9}{2} \text{ by symmetry}
\end{aligned}$$

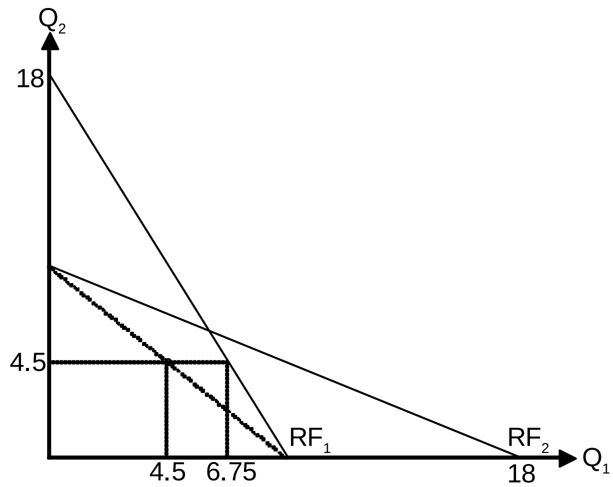
Total market output will be $Q = Q_1 + Q_2 = \frac{9}{2} + \frac{9}{2} = 9$ and prices will be $P = 20 - 9 = 11$. Profits will be $\pi_1 = \pi_2 = (11 - 2)\frac{9}{2} = \frac{81}{2}$. Total profits will be $\pi = \pi_1 + \pi_2 = 81$.

In contrast, if there was a monopoly, we would have $MR = 20 - 2Q$. The equilibrium conditions would be

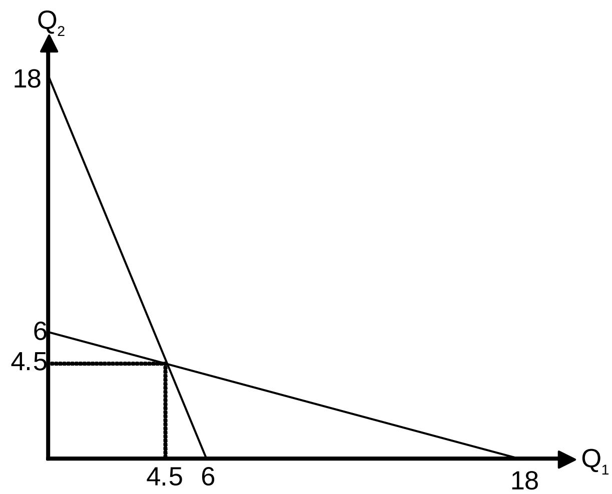
$$\begin{aligned}
MR &= MC \\
2 &= 20 - 2Q \\
2Q &= 18 \\
Q &= 9
\end{aligned}$$

Total output would be $Q = 9$. Prices would be $P = 20 - 9 = 11$. Total monopoly profit would then be $\pi = (11 - 2)\,9 = 81$. The two outcomes are the same.

Under the Cournot assumption, the cartel cannot be supported because by assuming that $\frac{\partial Q_i}{\partial Q_j} = 0\ \forall i \neq j$, we are assuming that no individual firm bears the full costs of cheating on the cartel. Therefore, each firm will have the individual incentive to cheat on the cartel, and it will thus break down. In the graph below, we see that the cartel output of $4\frac{1}{2}$ cannot be supported because it does not lie on the reaction curves. In fact, if $Q_2 = 4\frac{1}{2}$, it would be optimal for $Q_1 = 9 - \frac{1}{4}\left(\frac{9}{2}\right) = \frac{36-9}{4} = \frac{27}{4} \neq 4\frac{1}{2}$. This is depicted below:



Under the assumption that $\frac{\partial Q_i}{\partial Q_j} = +1 \; \forall i \neq j$, each firm knows that the other firm will fully react to its decision to increase (or decrease) output. As a result, any attempts to cheat on the cartel will be fully punished, so no firm will have the individual incentive to deviate. Graphically, this is so because the cartel output levels of $Q_1 = Q_2 = 4\frac{1}{2}$ lie on each firm's reaction curves:



2.4 Part (d)

Suppose we assume $\frac{\partial Q_i}{\partial Q_j} = -1 \; \forall i \neq j$. What is the outcome now? What is the intuition behind this outcome?

2.4.1 Answer

Intuitively, if $\frac{\partial Q_i}{\partial Q_j} = -1 \; \forall i \neq j$, then each firm will react in the opposite direction to a change in quantity produced by the other firm. A conversation between two firms with this property would go something like "If you don't produce this item, I will. If I don't produce it, you will." We would expect, then, that the only outcome compatible with such an attitude would be the competitive outcome where each producer is indifferent between producing and not producing. The analysis is as follows:

$$\begin{aligned}
\pi_1 &= (P - MC_1)\, Q_1 \\
&= (20 - Q_1 - Q_2 - 2)\, Q_1 \\
&= 18Q_1 - Q_1^2 - Q_1 Q_2
\end{aligned}$$

The first order conditions are:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial Q_1} &= 18 - 2Q_1 + Q_1 - Q_2 = 0 \\
Q_1 &= 18 - Q_2 \text{ Firm 1's reaction function} \\
Q_2 &= 18 - Q_1 \text{ Firm 2's reaction function by symmetry}
\end{aligned}$$

The two reaction functions are linearly dependent. That is, they do not yield any additional information. Therefore, there are infinite solutions of the form $Q_1 + Q_2 = 18$. In any case, $Q = Q_1 + Q_2 = 18$. $P = 20 - Q = 2 = MC_i$, $i \in \{1, 2\}$. This is exactly the result that would occur under perfect competition.

3 Question 3

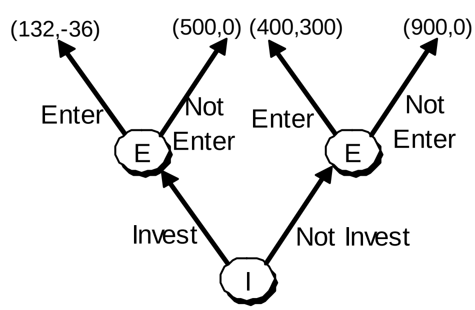
Consider the following entry game which is played sequentially. An incumbent firm will first choose between "invest" or "not invest." By investing in equipment the incumbent can lower its marginal cost of production. The potential entrant then chooses between "Enter" or "Not Enter." The payoffs are 900,0 if there is "not invest" and "not enter" (payoff for incumbent is listed first). Under "invest" and "not enter" the payoffs are 500,0. Under "not invest" and "enter" the payoffs are 400,300 and, finally, under "invest" and "enter" the payoffs are 132,-36

3.1 Part (a)

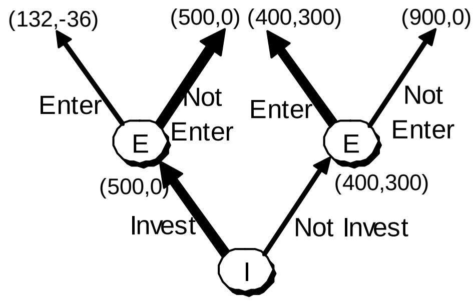
Illustrate this game in extensive form (tree diagram). What is the solution to this game?

3.1.1 Answer

The game in extensive form is:



Solving the game by backwards induction, we have:



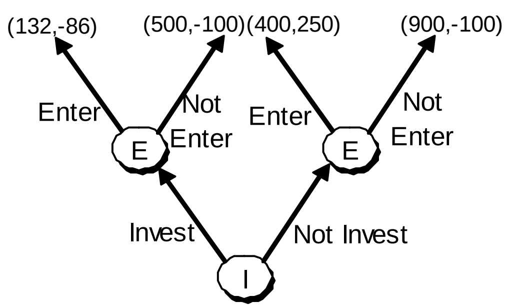
The solution is that the incumbent firm will invest and the potential entrant will stay out.

3.2 Part (b)

Suppose the potential entrant threatens to enter no matter what the incumbent chooses. Is this threat credible? Explain. How might the potential entrant make its threat credible? Give a brief explanation (one to two sentences).

3.2.1 Answer

Clearly this threat is not credible. If the incumbent invests, it will not be in the entrant's best interest to carry out his threat. The entrant would receive -36 if it entered and 0 if it stayed out. The entrant could make its threat credible by investing in some technology that could perhaps lower its marginal costs and change its payoffs in such a way that it is always in its best interest to enter. Such a situation is depicted below:



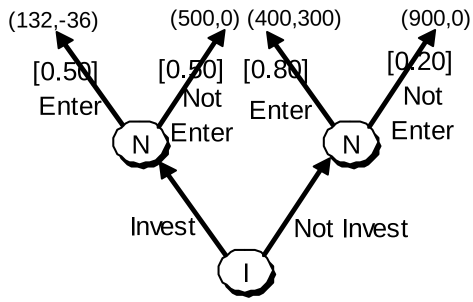
This technology costs 100 but lowers operating expenses by 50. With such technology, the entrant now has a credible threat to enter no matter what the incumbent does. This is a cop-out of sorts, however. First, it changes the order of the game. Now, it is actually the entrant who first chooses whether or not to invest in this hand-tying technology. In this Secondly, if we assume away this structural change of the game, it still changes the payoffs. In either case, it is actually a different game that we are analyzing.

3.3 Part (c)

Suppose the incumbent believes that the potential entrant goes through spells of "irrationality" in the sense that this firm may enter when it is not in its best interest to do so, or that it may choose not to enter when it is in its best interest to enter. More specifically, assume that the probability that the potential entrant enters when it is not in its best interest to do so is 0.50 and that the probability that the potential entrant does not enter when it is in its best interest to enter is 0.20. Assuming that the incumbent desires to maximize its expected payoff, what is the outcome of this game now? Show work.

3.3.1 Answer

This game now becomes a one-player game with the incumbent firm passing the move to one of two different nodes played by nature. I have drawn the extensive form of this game:



In some sense, the incumbent firm is choosing between one of two lotteries. By assumption, this firm is risk neutral, so it will choose the lottery with the greatest expected value. The two lotteries are:

Invest	Enter	Not Enter		Not Invest	Enter	Not Enter
p	0.50	0.50	and	p	0.80	0.20
π_I	132	500		π_I	400	900

The expected values are:

$$\begin{aligned} E(\pi_I|\text{Invest}) &= \frac{1}{2}132 + \frac{1}{2}500 \\ &= \frac{632}{2} \\ &= 216 \\ E(\pi_I|\text{Not Invest}) &= \frac{4}{5}400 + \frac{1}{5}900 \\ &= \frac{2500}{5} \\ &= 500 \end{aligned}$$

Given these choices and the assumption that all it cares about is the expected profits it will receive, the firm will choose not to invest. The entrant will enter with probability $\frac{1}{2}$ and not enter with probability $\frac{1}{2}$.