A Rational-Expectations Model of Goods Markets

Robert Gibbons, Richard Holden and Michael Powell*

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Abstract

We analyze a rational-expectations model of information acquisition and price formation in the spirit of Grossman and Stiglitz (1976, 1980), but for *goods markets*: markets where prices and net supply must be non-negative. Players have heterogeneous costs of acquiring information, and those who choose not to acquire information make inferences from the equilibrium price. We assume that all random variables in the model are uniformly distributed and independent of each other. Our model is tractable enough to yield closed-form solutions and is applicable to the markets for physical goods often analyzed in contract theory and international trade. (JEL: D80, G10.)

*Gibbons: Massachusetts Institute of Technology and NBER, rgibbons@mit.edu. Holden: Massachusetts Institute of Technology and NBER, rholden@mit.edu. Powell: Massachusetts Institute of Technology, mlp@mit.edu. We thank participants at MIT's Theory and Finance lunches for helpful comments and MIT Sloan's Program on Innovation in Markets and Organizations for financial support.

1 Introduction

Since at least Hayek (1945), economists have understood that prices can convey information, and hence that "We must look at the price system as ... a mechanism for communicating information if we want to understand its real function" (p. 526). Beginning in the 1970s, economists such as Grossman (1977) and Grossman and Stiglitz (1976, 1980) developed formal models analyzing this issue. In these classic models, some market participants make a costly investment in becoming informed about the value of an asset; others do not, but make rational inferences from the equilibrium price. The analysis is intricate because the price plays two roles: informing uninformed parties about the asset's value, but also clearing the market. This dual role for the price requires computation of the price function as a fixed point, and closed-form solutions are typically not available. One special case that does admit closed-form solutions is when agents have exponential utility functions and all random variables are jointly normally distributed.[1]

Many of these models of rational-expectations equilibrium seem best suited for (and are often intended as) models of financial markets, as opposed to goods markets, in two senses. First, models that assume normal distributions allow prices and quantities to be negative, which may occur through short-selling and other practices in some financial markets, but seem unfamiliar in goods markets (where free disposal keeps prices positive and the asset being a physical good keeps quantities positive). Second, models that require the presence of noise traders again may be consistent with financial markets but seem ill-suited for analyzing most goods markets (where the populations on both sides of the market are fairly stable, without the churning of noise traders who may quickly go bankrupt).

Another feature of the existing literature is that these models have not proven tractable enough to be embedded in larger applications, in order to include rational-expectations price formation in models of other economic phenomena. For example, if an application involves

[1] Examples include: Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Verrecchia (1982), Admati (1985), Wang (1993), Wang (1994), Veldkamp (2006), Chamley (2007), Yang and Ganguli (2008).

a cumulative distribution (say, in computing aggregate demand), then models involving normal distributions will not have closed-form solutions for such cumulative distributions, thus potentially prohibiting analysis of the application of interest.

In this paper we construct a rational-expectations model of price formation in goods markets. We assume uniform distributions with positive supports, so prices and quantities are always positive. And, to avoid having market participants who may quickly go bankrupt, we do not introduce noise traders within our focal industry, but instead assume that the good traded in our industry may also serve as an intermediate good in a second industry (where demand is random). The model delivers explicit solutions for both the price function and the proportion of agents who become informed.

The main technical difficulty in our analysis is that, unlike in Grossman and Stiglitz ("GS"), the equilibrium price function is not linear. We show, however, that the price function is piece-wise linear over three regions of parameter space. These three regions emerge naturally from our assumption of a uniform distribution on the asset's ultimate value, because uninformed traders' conditional belief about this value given the market price can take one of three forms: an upper tail of the prior distribution of asset values, a lower tail, or the entire support of possible values. Unlike GS, therefore, we find that the informativeness of the price mechanism depends on the realization of the price. In particular, the price mechanism is more informative at higher and lower prices (converging to perfectly informative at the maximum and minimum prices), but completely uninformative in a middle range of prices. This piece-wise linearity allows us to obtain explicit solutions, as well as perform comparative-statics exercises and revisit the seven conjectures from Grossman and Stiglitz (1980).

Our interest in this model is both direct and indirect. That is, we are interested in the model not only as a rational-expectations model of price formation in goods markets, but also as a model to be embedded in a larger application. In particular, in Gibbons, Holden and Powell (2009) ("GHP2"), we embed the present paper's model of price formation in a model of firms' integration decisions. Thus, the present paper allows us to expand the focus of the transaction-cost / property-rights literature on the boundary of the firm (e.g., Williamson (1971), Klein, Crawford and Alchian (1978), Grossman and Hart (1986)): rather than study the integration versus non-integration decision of one dyad in isolation, we analyze how the separate integration decisions of a market's worth of dyads interact through the market's pricing function. In this sense, in GHP2 we analyze *market* versus hierarchy, not non-integration versus integration.

The remainder of the paper is organized as follows. Section 2 states the problem, while section 3 characterizes the equilibrium and analyses comparative statics. Section 4 discusses related literature and concludes.

2 Statement of the Problem

There is a unit mass of players, indexed by $i \in [0,1]$. An intermediate good (a "widget") can be used to produce a final good. Each player is endowed with $w_i \in \{0,1\}$ widgets. The aggregate endowment of widgets is $y \leq 1$, and without loss of generality, let $w_i = 1$ if $i \leq y$.

At some cost c_i, a widget can be transformed into a final good that consumers value at v. This transformation cost is drawn from a uniform distribution on support $[0, \bar{c}]$. Consumers' value for a final good is uncertain, with v uniformly distributed on $[\underline{v}, \bar{v}]$. As in GS, players can pay to become informed about the value v. More precisely, before observing c_i, player i can pay a cost k_i to learn v (without error). We assume k_i to be uniformly distributed on $[0, \bar{k}]$.

Players not endowed with a widget may purchase one in the intermediate-good market, and players who are endowed may sell into the market. Additionally, there is a set of outside players of uncertain mass z, each of whom demands one widget inelastically. We think of this demand as coming from a "nearby" industry that also has use for widgets, and we assume that z is uniformly distributed on $[\underline{z}, \bar{z}]$. All the random variables in the model

(v, c, k and z) are independent of each other. Denote the market price for a widget as p. Players who are not directly informed about v (because they chose not to pay the cost k_i) make rational inferences about v from the equilibrium price.

Equilibrium in the market for widgets is determined by the price that equates supply (from both informed and uninformed sellers) and demand (from both informed and uninformed buyers). If this price perfectly revealed v there would be no incentive for any player to pay the cost k_i of learning v, but then prices would be uninformative. This contradiction is the GS non-existence result. To avoid this result there must not be a one-to-one mapping between prices and v. This is why we have a random outside demand for widgets, z.

2.1 Timing

To be more precise about the timing and assumptions, suppose there are five time periods. At the beginning of the first period ("information acquisition"), the value of the final good v is drawn from $U[\underline{v}, \bar{v}]$ and z is drawn from $U[\underline{z}, \bar{z}]$, both of which are unobserved. Players observe their private cost of becoming informed $k_i \sim U\left[0, \bar{k}\right]$ and decide whether or not to become informed.[2] In the second period ("endowment"), players learn their index i and all players with $i \leq y$ are endowed with $w_i = 1$ widgets.

In the third period ("price formation and trading"), player i observes $c_i \sim U[0, \bar{c}]$ and $\varphi_i \in \{\emptyset, v\}$, a signal about the value v of the final good. Let $\varphi_i = \emptyset$ denote the uninformative signal that obtains if k_i is not incurred in the first period and $\varphi_i = v$ the perfectly informative signal that obtains if k_i is incurred. Let $s_i = (c_i, \varphi_i)$ be the vector of i's signals in period one. Additionally, a mass z of outside parties each demand a single widget at any price $p \leq \bar{v}$. Knowing φ_i and facing price p, player i decides whether to buy/sell/keep a widget at price p.

In the fourth period ("production"), if player i has a widget, she can transform it into

[2] We assume that the players do not know their index, i, at this point to reduce the notational demands that arise if investment choices are conditioned on individual endowments. This assumption is otherwise inconsequential.

$q_i = 1$ final good at cost c_i. If player i does not have a widget, then $q_i = 0$.

In the fifth period ("sales and production"), final goods sell at price v. Gross payoffs are determined as follows

$$\pi_i(v, s_i, p) = \begin{cases} v - c_i - p + pw_i & \text{if transform widget into final good} \\ pw_i & \text{if not,} \end{cases}$$

so we can write $\pi_i(v, s_i, p) = (v - c_i - p)\, q_i + pw_i$.

Note that, as in GS and the ensuing literature, out model of price formation in the second period is not an extensive-form model of strategic decision-making, but rather a reduced-form model of price-taking behavior. Kyle (1985) and later papers depart from the GS tradition by allowing strategic trading behavior among agents who know that some may have differential information, but even these models (like those in the GS tradition) do not explore the possibility of strategic information transmission prior to the trading process. For example, one could allow cheap talk before trading, akin to the cheap talk before a double auction analyzed by Farrell and Gibbons (1989). Like the GS and Kyle traditions, here we also ignore the possibility of pre-trade strategic communication, but we note that our economic environment may be tractable enough to facilitate such an analysis.

3 Equilibrium

3.1 Rational-Expectations Equilibrium

In this subsection we take λ, the fraction of players who are informed, as given. We solve for the price function $p_\lambda(\cdot, \cdot)$ that both clears the market and communicates information to uninformed players. Informed players are willing to purchase (or keep) a widget and transform it into a final good ($q_i = 1$) if and only if $p + c_i \leq v$, so the largest value of c_i at which an informed player is willing to buy a widget is $c_I(v, p) = v - p$. Analogously, uninformed players are willing to purchase (or keep) and transform a widget iff $p + c_i \leq E[v | p_\lambda(\cdot, \cdot) = p]$, so we have $c_U(p) = E[v|p_\lambda(\cdot,\cdot) = p] - p$.

Let the net supply of widgets be denoted $x = y - z$, so that $x \sim U[\underline{x}, \bar{x}]$, where $\underline{x} = y - \bar{z}$ and $\bar{x} = y - \underline{z}$. Market-clearing requires that in each state of the world (x, v),

$$\lambda \frac{c_I(v,p)}{\bar{c}} + (1-\lambda)\frac{c_U(p)}{\bar{c}} = x, \tag{1}$$

or

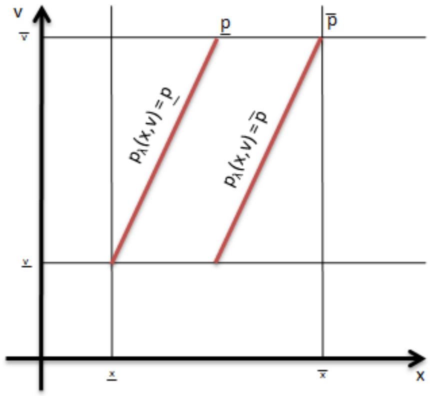
$$E[v|p_\lambda(\cdot,\cdot) = p] = \frac{1}{1-\lambda}(p + \bar{c}x - \lambda v). \tag{2}$$

As in GS, note that computing the conditional expectation $E[v|p_\lambda(\cdot,\cdot) = p]$ requires knowing the *price function* $p_\lambda(\cdot,\cdot)$, not merely the realized *price* p.

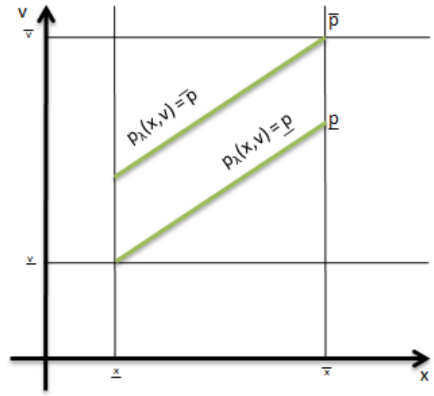
Definition 1 *Assume a fraction λ of the players is informed. A* ***rational-expectations equilibrium (REE)*** *is a price function $p_\lambda(x,v)$ and an allocation $\{q_i^*\}_{i\in\{0,1\}}$ such that*

1. $q_i^* \in \operatorname{argmax}_{q_i \in \{0,1\}} E_{x,v}[\pi_i(v, s_i, p)|\, p_\lambda(\cdot,\cdot) = p, s_i]$
2. (1) *holds for all x, v.*

We now establish the existence of an REE by construction. The price function is a mapping $p : [\underline{x}, \bar{x}] \times [\underline{v}, \bar{v}] \to \mathbb{R}$. There are two cases we (may) need to consider, depending on the parameters (endogenous and exogenous). Let $\bar{p} = p_\lambda(\bar{x}, \bar{v})$ and $\underline{p} = p_\lambda(\underline{x}, \underline{v})$. Note that the highest equilibrium price will be $p_H = p_\lambda(\underline{x}, \bar{v}) > \max\{\underline{p}, \bar{p}\}$ and the lowest will be $p_L = p_\lambda(\bar{x}, \underline{v}) < \max\{\underline{p}, \bar{p}\}$. Thus, as we will see, $\bar{p}$ and $\underline{p}$ are the boundary prices between the pieces of the piecewise-linear price function, not the highest and lowest values of that function. As we will show below, if $\bar{p} \leq \underline{p}$, we say that we are in case A, and if $\bar{p} > \underline{p}$, we say that we are in case B.



Case A



Case B

For now assume that we are in case A. We will need to verify below that this is indeed true. Define

$$
\begin{aligned}
R_\lambda^1 &= \{(x,v) : p_\lambda(x,v) \le \bar{p}\} \\
R_\lambda^2 &= \left\{(x,v) : \bar{p} < p_\lambda(x,v) \le \underline{p}\right\} \\
R_\lambda^3 &= \left\{(x,v) : \underline{p} < p_\lambda(x,y)\right\}.
\end{aligned}
$$

We now conjecture that there is a piecewise-linear price function

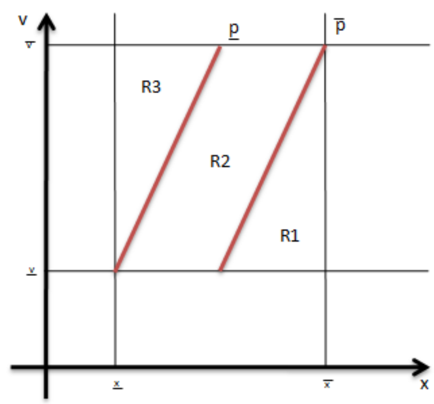
$$
\begin{aligned}
p_\lambda(x,v) &= 1_{\left\{(x,v)\in R_\lambda^1\right\}}\left[\beta_0^1 + \beta_1^1 v - \beta_2^1 x\right] + 1_{\left\{(x,v)\in R_\lambda^2\right\}}\left[\beta_0^2 + \beta_1^2 v - \beta_2^2 x\right] + \\
&\quad 1_{\left\{(x,v)\in R_\lambda^3\right\}}\left[\beta_0^3 + \beta_1^3 v - \beta_2^3 x\right] \\
&\equiv 1_{\left\{(x,v)\in R_\lambda^1\right\}} p_\lambda^1(x,v) + 1_{\left\{(x,v)\in R_\lambda^2\right\}} p_\lambda^3(x,v) + 1_{\left\{(x,v)\in R_\lambda^3\right\}} p_\lambda^3(x,v), \qquad (3)
\end{aligned}
$$

which is a fixed point of the equation

$$
\mu_{v|p} \equiv E\left[v \mid p_\lambda(\cdot,\cdot) = p_\lambda(x,v)\right] = \frac{1}{1-\lambda}\left(p_\lambda(x,v) + \bar{c}x - \lambda v\right), \qquad (4)
$$

and we seek to solve for the parameter vectors β^1, β^2, and β^3.

It is important to note that, since individuals form their conditional expectations of v according to the equilibrium price function, the form of the conditional expectation depends on which region the price lies in: if $p \leq \bar{p}$, then individuals know that the price was determined by the β^1 coefficients; if $\bar{p} < p \leq \underline{p}$, then individuals know that the price was determined by the β^2 coefficients; and if $\underline{p} < p$, then individuals know that the price was determined by the β^3 coefficients. We now analyze these regions in turn.



Case A Regions

Region 1

If $p \leq \bar{p}$, then individuals believe that $v|\, p_\lambda(\cdot,\cdot) = p \sim U\left[\underline{v}^1(p), \bar{v}^1(p)\right]$, where $\underline{v}^1(p)$ and $\bar{v}^1(p)$ are respectively the lowest and highest values of v consistent with the realized price level p. It can be shown that $\underline{v}^1(p) = \underline{v}$ and $\bar{v}(p)$ solves $p^1_\lambda(\bar{x}, \bar{v}^1(p)) = p$ or $\bar{v}^1(p) = \frac{1}{\beta^1_1}\left(p - \beta^1_0 + \beta^1_2 \bar{x}\right)$. Given a price $p \leq \bar{p}$, the conditional expectation of v is then

$$\begin{aligned} \mu^1_{v|p} &\equiv E\left[v|\, p_\lambda(\cdot,\cdot) = p\right] \\ &= \frac{\underline{v}^1(p) + \bar{v}^1(p)}{2} = \frac{\underline{v} + \left(1/\beta^1_1\right)\left(p - \beta^1_0 + \beta^1_2 \bar{x}\right)}{2}. \end{aligned}$$

Market-clearing in (2) then requires that

$$\frac{1}{2}\left[\underline{v}+\frac{1}{\beta_1^1}\left(\underbrace{\left(\beta_0^1+\beta_1^1 v-\beta_2^1 x\right)}_{p_\lambda^1(x,v)}-\beta_0^1+\beta_2^1\bar{x}\right)\right]\equiv\frac{1}{1-\lambda}\left(\underbrace{\left(\beta_0^1+\beta_1^1 v-\beta_2^1 x\right)}_{p_\lambda^1(x,v)}+\bar{c}x-\lambda\right),$$

where the equivalence relation reminds us that this holds as an identity. Solving for β_0^1, β_1^1, and β_2^1, we obtain

$$\begin{aligned}
\beta_0^1 &= (1-\lambda)\frac{\underline{v}+(\bar{c}/\lambda)\,\bar{x}}{2}, \\
\beta_1^1 &= \frac{1+\lambda}{2}, \text{ and} \\
\beta_2^1 &= \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}.
\end{aligned}$$

Region 2

If $\bar{p} < p \leq \underline{p}$, then individuals believe that $v|\,p_\lambda(\cdot,\cdot) = p \sim U\left[\underline{v}^2(p), \bar{v}(p)\right]$, where $\underline{v}^2(p) = \underline{v}$ and $\bar{v}^2(p) = \bar{v}$, so that

$$\mu_{v|p}^2 = \frac{\underline{v}^2(p)+\bar{v}^2(p)}{2} = \frac{\underline{v}+\bar{v}}{2}.$$

Market-clearing in (2) then requires that

$$\frac{\underline{v}+\bar{v}}{2}\equiv\frac{1}{1-\lambda}\left(\underbrace{\beta_0^2+\beta_1^2 v-\beta_2^2 x}_{p_\lambda^2(x,v)}+\bar{c}x-\lambda v\right).$$

Solving for β_0^2, β_1^2, and β_2^2 we obtain

$$\begin{aligned}
\beta_0^2 &= (1-\lambda)\frac{\underline{v}+\bar{v}}{2}, \\
\beta_1^2 &= \lambda, \text{ and} \\
\beta_2^2 &= \bar{c}.
\end{aligned}$$

Region 3

Finally, if $\underline{p} < p$, then individuals believe that $v|\, p_\lambda(\cdot,\cdot) = p \sim U\left[\underline{v}^3(p), \bar{v}^3(p)\right]$, where $\underline{v}^3(x,v)$ solves $p^3_\lambda\left(\underline{x}, \underline{v}^3(x,v)\right) = p$ or $\underline{v}(p) = \frac{1}{\beta_1}\left(p - \beta^3_0 + \beta^3_2 \underline{x}\right)$, so that

$$\mu^3_{v|p} = \frac{\underline{v}^3(p) + \bar{v}^3(p)}{2} = \frac{\left(1/\beta^3_1\right)\left(p - \beta^3_0 + \beta^3_2 \underline{x}\right) + \bar{v}}{2}.$$

Market-clearing in (2) then requires that

$$\frac{1}{2}\left[\frac{1}{\beta^3_1}\left(\underbrace{\beta^3_0 + \beta^3_1 v - \beta^3_2 x}_{p^3_\lambda(x,v)} - \beta^3_0 + \beta^3_2 \underline{x}\right) + \bar{v}\right] \equiv \frac{1}{1-\lambda}\left(\underbrace{\beta^3_0 + \beta^3_1 v - \beta^3_2 x}_{p^3_\lambda(x,v)} + \bar{c}x - \lambda v\right).$$

Solving for β^3_0, β^3_1, and β^3_2 we obtain

$$\begin{aligned}
\beta^3_0 &= (1-\lambda)\frac{(\bar{c}/\lambda)\,\underline{x} + \bar{v}}{2}, \\
\beta^3_1 &= \frac{1+\lambda}{2}, \text{ and} \\
\beta^3_2 &= \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}.
\end{aligned}$$

To summarize, we have the following

$$\begin{aligned}
p^1_\lambda(x,v) &= (1-\lambda)\frac{\underline{v} + (\bar{c}/\lambda)\,\bar{x}}{2} + \frac{1+\lambda}{2}v - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x, \\
p^2_\lambda(x,v) &= (1-\lambda)\frac{\underline{v} + \bar{v}}{2} + \lambda v - \bar{c}x, \\
p^3_\lambda(x,v) &= (1-\lambda)\frac{(\bar{c}/\lambda)\,\underline{x} + \bar{v}}{2} + \frac{1+\lambda}{2}v - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x.
\end{aligned} \tag{5}$$

By construction, our conjectured price function (3) is a fixed point of the market-clearing equation (4), where

$$\mu_{v|p}(x,v) = \begin{cases} \frac{\underline{v}+\bar{v}}{2} - \frac{\bar{v}-\underline{v}}{2} + \frac{(\bar{c}/\lambda)(\bar{x}-x)}{2} & \text{if} \quad p_\lambda(x,v) \leq \bar{p} \\ \frac{\underline{v}+\bar{v}}{2} & \text{if} \quad \bar{p} < p_\lambda(x,v) \leq \underline{p} \\ \frac{\underline{v}+\bar{v}}{2} + \frac{v-\underline{v}}{2} - \frac{(\bar{c}/\lambda)(x-\underline{x})}{2} & \text{if} \quad \underline{p} < p_\lambda(x,v). \end{cases}$$

It is reassuring to note that $\beta_0^1+\beta_1^1\bar{v}-\beta_2^1\bar{x} = \beta_0^2+\beta_1^2\bar{v}-\beta_2^2\bar{x}$ and $\beta_0^2+\beta_1^2\underline{v}-\beta_2^2\underline{x} = \beta_0^3+\beta_1^3\underline{v}-\beta_2^3\underline{x}$, so that the choice of how to deal with the boundaries of the regions is immaterial.

Of course, all of this was derived under the assumption that we are in case A, so we now must check that $\bar{p} \leq \underline{p}$, or

$$\bar{p} = (1-\lambda)\frac{(\underline{v}+(\bar{c}/\lambda)\bar{x})}{2} + \frac{1+\lambda}{2}\bar{v} - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}\bar{x} \leq (1-\lambda)\frac{\underline{v}+\bar{v}}{2} + \lambda\underline{v} - \bar{c}\underline{x} = \underline{p}.$$

Rearranging, we require

$$\lambda \leq \bar{c}\frac{\bar{x}-\underline{x}}{\bar{v}-\underline{v}} \equiv \bar{c}\frac{\sigma_x}{\sigma_v}, \tag{6}$$

where $\sigma_x = \frac{\bar{x}-\underline{x}}{\sqrt{12}}$ and $\sigma_v = \frac{\bar{v}-\underline{v}}{\sqrt{12}}$. Condition (6) is equivalent to $\frac{\bar{c}}{\lambda} \geq \frac{\sigma_v}{\sigma_x}$, which can be interpreted graphically as follows. The slope of the iso-price lines $p_\lambda(x,v) = p$ in the graphs above is $\frac{\bar{c}}{\lambda}$. The slope of the line connecting the lower left corner $(\underline{x},\underline{v})$ to the upper right corner $(\bar{x},\bar{v})$ is $\frac{\sigma_v}{\sigma_x}$. Whenever the slope of the iso-price lines exceeds the slope of the diagonal, we are in case A; otherwise, we are in case B. Thus, there is a nontrivial region of the endogenous parameter space for which the price function (5) is indeed a valid fixed point. In fact, one can ensure that we are *always* in the case A by making the exogenous parameter restriction $\bar{c}\frac{\sigma_x}{\sigma_v} \geq 1$. Then, since $\lambda \leq 1$, (6) always holds.

The analysis of case B follows identical reasoning and is thus relegated to the appendix. We note, however, that in certain environments the appropriate assumptions on exogenous variables may be such that one is in case B. In particular, the limiting case in which prices become perfectly informative (away from the extreme prices p_L and p_H) lies within the domain of case B. While the analysis is identical, the expression for expected utilities differ (as the appendix makes clear).

It is also worth noting that $p_\lambda(x,v)$ is continuous at $\lambda = \bar{c}\frac{\sigma_x}{\sigma_v}$ and hence everywhere continuous in λ. It is also everywhere continuous in (x,v) as it is piecewise linear. Finally, we provide conditions under which prices are positive–a key requirement of goods markets with free disposal. Since prices are strictly increasing in v and strictly decreasing in x, prices are lowest at $(\bar{x},\underline{v})$ (i.e. when net supply is high and informed demand is low). Since $(\bar{x},\underline{v})$ occurs in R_1, we have that

$$p_\lambda(\bar{x},\underline{v}) = p_\lambda^1(\bar{x},\underline{v}) = \underline{v} - \bar{c}\bar{x},$$

so that $p_\lambda(\bar{x},\underline{v}) > 0$ if and only if $\underline{v} > \bar{c}\bar{x}$. Thus, when $\underline{v} > \bar{c}\bar{x}$, prices are everywhere positive. All these facts are collected in the following theorem.

Theorem 1 *Given λ, there exists an REE characterized by a price-wise linear price function*

$$p_\lambda(x,v) \equiv 1_{\{(x,v)\in R_\lambda^1\}} p_\lambda^1(x,v) + 1_{\{(x,v)\in R_\lambda^2\}} p_\lambda^3(x,v) + 1_{\{(x,v)\in R_\lambda^3\}} p_\lambda^3(x,v),$$

where $p_\lambda^j(x,v)$ is linear in (x,v) for all j. Further, when $\lambda \le \bar{c}\frac{\sigma_x}{\sigma_v}$,

$$\begin{aligned}
p_\lambda^1(x,v) &= (1-\lambda)\frac{\underline{v} + (\bar{c}/\lambda)\bar{x}}{2} + \frac{1+\lambda}{2}v - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x, \\
p_\lambda^2(x,v) &= (1-\lambda)\frac{\underline{v} + \bar{v}}{2} + \lambda v - \bar{d}x, \\
p_\lambda^3(x,v) &= (1-\lambda)\frac{(\bar{c}/\lambda)\underline{x} + \bar{v}}{2} + \frac{1+\lambda}{2}v - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x,
\end{aligned}$$

and when $\lambda > \bar{c}\frac{\sigma_x}{\sigma_v}$,

$$\begin{aligned}
p_\lambda^1(x,v) &= (1-\lambda)\frac{\underline{v} + (\bar{c}/\lambda)\bar{x}}{2} + \frac{1+\lambda}{2}v - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x, \\
p_\lambda^2(x,v) &= (1-\lambda)\frac{\bar{c}}{\lambda}\frac{\underline{x} + \bar{x}}{2} + v - \frac{\bar{c}}{\lambda}x, \\
p_\lambda^3(x,v) &= (1-\lambda)\frac{(\bar{c}/\lambda)\underline{x} + \bar{v}}{2} + \frac{1+\lambda}{2}v - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x.
\end{aligned}$$

Additionally, $p_\lambda(x,v)$ is continuous and, when $\underline{v} > \bar{c}\bar{x}$, prices are everywhere positive.

3.2 Full Equilibrium

We now turn to endogenizing information acquisition. Recall that, prior to observing the production cost (c_i) or the price (p), each player can, at cost k_i, learn the value of the final good to consumers (v). Given λ, gross of information-acquisition costs, an informed player receives expected payoff

$$\begin{aligned}\Pi^I(\lambda) &\equiv E_{x,v,c_i}\left[\pi_i(v,s_i,p)|\, p_\lambda(\cdot,\cdot)=p_\lambda(x,v), c_i, \varphi_i = v\right] \\ &= \int_v \left[\int_x \left[\int_{c_i} (v-p_\lambda(x,v)-c_i)\, 1_{\{c_i<v-p_\lambda(x,v)\}} \frac{1}{\bar{c}} dc_i\right] \frac{1}{\bar{x}-\underline{x}} dx\right] \frac{1}{\bar{v}-\underline{v}} dv \\ &= \frac{1}{\bar{x}-\underline{x}} \frac{1}{\bar{v}-\underline{v}} \frac{1}{2\bar{c}} \int_v \left[\int_x (v-p_\lambda(x,v))^2\, dx\right] dv. \qquad (7)\end{aligned}$$

Given λ, an uninformed player receives expected payoff

$$\begin{aligned}\Pi^U(\lambda) &\equiv E_{x,v,c_i}\left[\pi_i(v,s_i,p)|\, p_\lambda(\cdot,\cdot)=p_\lambda(x,v), c_i, \varphi_i = \emptyset\right] \\ &= \int_v \left[\int_x \left[\int_{c_i} (v-p_\lambda(x,v)-c_i)\, 1_{\left\{c_i<\mu_{v|p}-p_\lambda(x,v)\right\}} \frac{1}{\bar{c}} dc_i\right] \frac{1}{\bar{x}-\underline{x}} dx\right] \frac{1}{\bar{v}-\underline{v}} dv \\ &= \frac{1}{\bar{x}-\underline{x}} \frac{1}{\bar{v}-\underline{v}} \frac{1}{2\bar{c}} \int_v \left[\int_x \left[\left(2v\mu_{v|p} - \left(\mu_{v|p}\right)^2 - 2vp_\lambda(x,v) + p_\lambda(x,v)^2\right)\right] dx\right] dv \qquad (8)\end{aligned}$$

As described in Claim 1 (contained in the appendix), depending whether or not $\lambda \leq \bar{c}\frac{\sigma_x}{\sigma_v}$, the expressions for $\mu_{v|p}(x,v)$ and $p_\lambda(x,v)$ are determined based on the pricing function from case A or case B. A player will become informed if the expected benefits of doing so exceed the private costs

$$\Delta(\lambda) \equiv \Pi^I(\lambda) - \Pi^U(\lambda) \geq k_i.$$

In words, in period one, each player will consider all possible states of the world (x, v, c_i) that may arise, the price of the widget that arises in each state, her expectations of v (if uninformed) given that particular price, and the cost of the decision making error that arises from producing (or not) a final good without using all the information. The expected cost of such decision-making errors is then compared to the private cost of becoming informed.

A full equilibrium is defined as follows.

Definition 2 *A **full equilibrium** is a fraction λ^*, a price function $p_\lambda(\cdot,\cdot)$, and an allocation $\{q_i^*\}_{i\in\{0,1\}}$ such that*

1. *A fraction λ^* of the parties optimally choose to become informed.*
2. $q_i^* \in \operatorname{argmax}_{q_i\in\{0,1\}} E_{x,v}\left[\pi_i(v,s_i,p)\middle|\, p_\lambda(\cdot,\cdot)=p, s_i\right]$
3. (1) *holds for all x, v.*

The remaining task is to compute the difference in expected utility $\Delta(\lambda)$ for both case A and case B. In the following subsection, we compute $\Delta(\lambda)$ for case A, relegating the computation for case B to the appendix.

3.2.1 Expected Utility Difference

For the remainder of this subsection, let us assume that we are in case A. The *ex ante* expected utility is a triple integral over (v, x, c_i) space, as in (7) and (8), so the expressions for $\Pi^I(\lambda)$ and $\Pi^U(\lambda)$ are quite messy, but the expression for the difference between the two, $\Delta(\lambda)$, is not. Since widget-purchase decisions follow a cutoff rule in c_i, we can integrate over c_i first as in (7) and (8). Subtraction then shows that the gains to becoming informed are

$$\Delta(\lambda) = \Pi^I(\lambda) - \Pi^U(\lambda) = \frac{1}{\bar{x}-\underline{x}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{2\bar{c}}\int_v\int_x \left(v-\mu_{v|p}(x,v)\right)^2 dx dv. \qquad (9)$$

As we are in case A, for each given v, all three regions of the (x, v) space are feasible for certain ranges of x. Let $R_1(v), R_2(v)$ and $R_3(v)$ denote these regions:

$$R_j(v) = \{x : (x,v) \in R_j\}.$$

It can be shown that

$$
\begin{aligned}
R_1(v) &= \{x : (x,v) \in R_1\} = \left(\bar{x} - \frac{\lambda}{\bar{c}}(\bar{v} - v), \bar{x}\right), \\
R_2(v) &= \{x : (x,v) \in R_2\} = \left(\underline{x} + \frac{\lambda}{\bar{c}}(v - \underline{v}), \bar{x} - \frac{\lambda}{\bar{c}}(\bar{v} - v)\right), \text{ and} \\
R_3(v) &= \{x : (x,v) \in R_3\} = \left(\underline{x}, \underline{x} + \frac{\lambda}{\bar{c}}(v - \underline{v})\right).
\end{aligned}
$$

The expression (9) can then be integrated over the three regions as follows:

$$
\begin{aligned}
\Delta(\lambda) &= \frac{1}{\bar{x} - \underline{x}} \frac{1}{\bar{v} - \underline{v}} \frac{1}{2\bar{c}} \int_{\underline{v}}^{\bar{v}} \left[\sum_{j=1}^{3} \int_{R_j(v)} \left(v - \mu_{v|p}^{j}(x,v)\right)^2 dx \right] dv \\
&= \frac{E_{x,v}\left[\sigma_{v|p}^2\right]}{2\bar{c}} = \frac{\sigma_v^2}{2\bar{c}} \left(1 - \frac{\lambda}{2} \frac{\sigma_v/\sigma_x}{\bar{c}}\right),
\end{aligned}
$$

where it is important to note that in region j, the conditional mean is given by $\mu_{v|p}^{j}(x,v)$. The last equality is derived in the appendix. If we define $K(\lambda) = \lambda \frac{\sigma_v/\sigma_x}{\bar{c}}$, we have that the returns to becoming informed are

$$
\Delta(\lambda) = \frac{\sigma_v^2}{2\bar{c}} \left(1 - \frac{1}{2} K(\lambda)\right),
$$

whenever $K(\lambda) \leq 1$ (which is the condition guaranteeing that we are in case A).

Proceeding similarly, when $K(\lambda) > 1$ we are in case B and as shown in the appendix, the difference in expected utilities is then given by

$$
\Delta(\lambda) = \frac{\sigma_v^2}{2\bar{c}} \left(\frac{1}{K(\lambda)}\right)^2 \left(1 - \frac{1}{2} \frac{1}{K(\lambda)}\right).
$$

In summary,

$$\Delta(\lambda) = \begin{cases} \frac{\sigma_v^2}{2\bar{c}}\left(1 - \frac{1}{2}K(\lambda)\right) & \text{if} \quad K(\lambda) \leq 1 \\ \frac{\sigma_v^2}{2\bar{c}}\left(\frac{1}{K(\lambda)}\right)^2\left(1 - \frac{1}{2}\frac{1}{K(\lambda)}\right) & \text{if} \quad K(\lambda) > 1. \end{cases}$$

3.2.2 Information Acquisition

Since profits are separable in the costs of becoming informed, any full equilibrium with an interior value of λ must involve a cutoff value k^* such that a player with information acquisition costs $k_i > k^*$ remains uninformed and a player with $k_i \leq k^*$ chooses to become informed. Given the assumption that $k \sim U\left[0, \bar{k}\right]$, the fraction of parties that is informed is then $\lambda^*(k^*) = \frac{k^*}{\bar{k}}$. This value k^* then solves $\Delta(\lambda^*(k^*)) = k^*$ or

$$\Delta(\lambda^*) = \lambda^*\bar{k}.$$

If we can show that $\Delta(\lambda)$ is continuous in λ and weakly decreasing, then the existence and uniqueness of such a λ^* is guaranteed. The following two lemmas show that these two conditions hold.

Lemma 1 *For $\sigma_x, \sigma_v > 0$, $\Delta(\lambda)$ is continuous in λ on $[0,1]$. $\Delta(\lambda)$ is not continuous at $\lambda = 0$ if $\sigma_x = 0$.*

Proof. *Clearly, $\Delta(\lambda)$ is continuous for all λ satisfying $K(\lambda) < 1$ and $K(\lambda) > 1$. It remains to show that $\Delta(\lambda)$ is continuous at $\tilde{\lambda}$ which solves $K\left(\tilde{\lambda}\right) = 1$. For this, all we need to show is that*

$$\lim_{\lambda \downarrow \tilde{\lambda}} \frac{\sigma_v^2}{2\bar{c}}\left(\frac{1}{K(\lambda)}\right)^2\left(1 - \frac{1}{2}\frac{1}{K(\lambda)}\right) = \frac{\sigma_v^2}{2\bar{c}}\left(1 - \frac{1}{2}K(\lambda)\right).$$

We know that

$$\begin{aligned}\lim_{\lambda \downarrow \tilde{\lambda}} \frac{\sigma_v^2}{2\bar{c}} \left(\frac{1}{K(\lambda)}\right)^2 \left(1 - \frac{1}{2}\frac{1}{K(\lambda)}\right) &= \frac{\sigma_v^2}{2\bar{c}} \left(\frac{1}{K\left(\tilde{\lambda}\right)}\right)^2 \left(1 - \frac{1}{2}\frac{1}{K\left(\tilde{\lambda}\right)}\right) \\ &= \frac{\sigma_v^2}{2\bar{c}}\left(1 - \frac{1}{2}\right) = \frac{\sigma_v^2}{2\bar{c}}\left(1 - \frac{1}{2}K\left(\tilde{\lambda}\right)\right),\end{aligned}$$

which establishes the first claim. To establish the second claim, take the following two sequences

$$\begin{aligned}\{\lambda_n, \sigma_{x,n}\} &= \left\{\frac{\bar{c}}{\sigma_v}\frac{1}{n}, \frac{1}{n}\right\} \\ \{\lambda_n, \sigma_{x,n}\} &= \left\{\frac{\bar{c}}{\sigma_v}\frac{2}{n}, \frac{1}{n}\right\}.\end{aligned}$$

Both sequences converge to $\lambda = \sigma_x = 0$. Along the first sequence, we have that $\lim_{n\to\infty} \Delta(\lambda_n) = \frac{\sigma_v^2}{4\bar{c}}$ and along the second sequence, we have that $\lim_{n\to\infty} \Delta(\lambda_n) = \frac{3\sigma_v^2}{32\bar{c}}$. Thus, there is a discontinuity at $\lambda = \sigma_x = 0$. ■

Lemma 2 *For $\sigma_x, \sigma_v > 0$, $\Delta(\lambda)$ is weakly decreasing in λ for all λ.*

Proof. *Since $\frac{\sigma_v^2}{2\bar{c}}\left(1 - \frac{1}{2}K(\lambda)\right)$ is linear in λ with a negative coefficient, $\Delta(\lambda)$ is strictly decreasing in λ for all λ such that $K(\lambda) \leq 1$. Next, suppose $K(\lambda) > 1$. Then define $\omega = \lambda^{-1}$.*

$$\frac{\partial}{\partial \omega}\Delta\left(\omega^{-1}\right) = \bar{c}\sigma_x^2\left(\omega - \frac{3}{4}\bar{c}\omega^2\frac{\sigma_x}{\sigma_v}\right).$$

Thus, $\frac{\partial}{\partial\omega}\Delta(\omega^{-1}) > 0$ if $\frac{\lambda}{\bar{c}} = \frac{1}{\omega\bar{c}} > \frac{3}{4}\frac{\sigma_x}{\sigma_v}$, which holds since $K(\lambda) > 1$ (and thus $\frac{\lambda}{\bar{c}} > \frac{\sigma_x}{\sigma_v}$). Since $\Delta(\omega^{-1})$ is increasing in $\omega = \frac{1}{\lambda}$, it is decreasing in λ. Therefore, $\Delta(\lambda)$ is decreasing in λ for all λ. ■

If $\Delta(0) \leq 0$, then it does not pay to become informed, so $\lambda^* = 0$ in the full equilibrium. Similarly, if $\lambda(1) \geq \bar{k}$, then it pays to become informed even if everyone else is informed, so $\lambda^* = 1$ in the full equilibrium.

Theorem 2 *For all* $\bar{k}, \bar{c}, \sigma_x$ *and* σ_v *strictly positive, there exists a full equilibrium. If* $\Delta(0) \leq 0$*, then* $\lambda^* = 0$ *and if* $\Delta(1) \geq \bar{k}$*, then* $\lambda^* = 1$*. Otherwise, there exists an interior solution to* $\Delta(\lambda^*) = \lambda^* \bar{k}$*, which is unique. For* $\sigma_x = 0$*, an equilibrium fails to exist.*

Proof. *From Lemma 1 and Lemma 2, we know that if* $\sigma_x > 0$*, then* $\Delta(\lambda)$ *is continuous and decreasing in* λ*. Thus,* $\Delta(\lambda) - \lambda\bar{k}$ *is continuous and strictly decreasing in* λ*. If* $\Delta(0) > 0$ *and* $\Delta(1) < \bar{k}$*, then* $\Delta(\lambda) - \lambda\bar{k} = 0$ *for some* $\lambda^* \in (0,1)$ *by the intermediate value theorem. By strict monotonicity, this* λ^* *is unique.*

For $\sigma_x = 0$*, for all* $\lambda > 0$*,* $\Delta(\lambda) = 0$*. Since* $\Delta(\lambda)$ *is discontinuous at* $\lambda = 0$*, it must be that* $\Delta(0) \neq 0$*. Therefore, since* $\bar{k}\lambda = 0$ *at* $\lambda = 0$ *and* $\bar{k}\lambda > 0$ *for all* $\lambda > 0$*, there is no* λ^* *such that* $\Delta(\lambda^*) = \bar{k}\lambda^*$*.* ■

As a special case, let us assume $\bar{c}\frac{\sigma_x}{\sigma_v} = 1$, so that we are in case A. Then either λ^* solves

$$\frac{\sigma_v^2}{2\bar{c}}\left(1 - \frac{\lambda^*}{2}\frac{\sigma_v/\sigma_x}{\bar{c}}\right) = \lambda^*\bar{k}$$

or $\lambda^* = 1$. That is,

$$\lambda^* = \min\left\{\frac{2\bar{c}\sigma_v^2}{\sigma_v^2\frac{\sigma_v}{\sigma_x} + 4\bar{c}^2\bar{k}}, 1\right\}$$

More generally, as shown in the appendix, $\Delta(\lambda)$ is increasing in σ_v and σ_x, so we can conclude that λ^* is increasing in σ_v, σ_x. Additionally, we know that $\Delta(\lambda) - \bar{k}\lambda$ is decreasing in $\bar{k}$, so λ^* is decreasing in $\bar{k}$. The effect of $\bar{c}$ on λ^* is indeterminate.

3.3 Comparative Statics

Grossman and Stiglitz (1980) begin their paper by making seven conjectures, saying that they may or may not be true in general, and then develop their CARA-Normal model in which all seven are indeed true. We now discuss these seven conjectures in the context of our model.

Conjecture 1 (GS 1) *The more players who are informed, the more informative is the* *price mechanism.*

GS use the Blackwell criterion to order information systems. That criterion is not applicable here, as it does not order uniform distributions with different supports. One simple way to proceed is to define the informativeness of the price mechanism as the expected reduction in the variance of v that results from observing p. We then have that

$$E_{x,v}\left[\sigma_v^2 - \sigma_{v|p}^2\right] = \lambda \frac{\sigma_v^2}{2} \frac{\sigma_v/\sigma_x}{\bar{c}},$$

which is clearly increasing in λ. Alternatively, Lehmann's criterion for monotone decision problems does order uniform distributions with differing support (Lehmann (1988)) and can be used to produce an analogous result.

Conjecture 2 (GS 2) *The more players who are informed, the lower the ratio of the expected utility of the informed to the uninformed.*

For GS, the ratio of the expected utility of the informed to the uninformed plays an integral role in computing the fraction of parties who are informed, and this conjecture provides the monotonicity required to ensure the existence and uniqueness of a solution. In our paper, the difference (rather than the ratio) in the expected utilities of the informed and the uninformed plays the analogous role, and this difference, $\Delta(\lambda)$, is shown to be decreasing in Lemma 2.

Conjecture 3 (GS 3) *The higher the cost of information, the smaller is the number of players who are informed in equilibrium.*

Our model features heterogeneous costs of acquiring information, so we need to define what it means for the cost of information to increase. If we define an increase in the cost of information acquisition as a first-order stochastic dominant shift in the distribution of information costs, then for our model, this corresponds to increasing $\bar{k}$. As argued in the previous section, we know that $\Delta(\lambda) - \bar{k}\lambda$ is decreasing in $\bar{k}$, so the solution to $\Delta(\lambda^*) - \bar{k}\lambda^* = 0$ is decreasing in $\bar{k}$.

Conjecture 4 (GS 4) *If the quality of information of the informed players increases then the price system becomes more informative but the equilibrium number of informed traders may increase or decrease.*

In our model if a player pays k_i she becomes perfectly informed with probability one. We have analyzed an extension in which, instead, she becomes informed with probability r. An increase in the quality of information of the informed can be thought of as an increase in r (the probability of becoming informed). In this extension, a party becomes informed if

$$\Delta(\lambda) \geq \frac{k_i}{r}.$$

The condition for an (interior) equilibrium is then $\Delta(\lambda^*) - \frac{\bar{k}}{r}\lambda^* = 0$. The quality of information plays the opposite role as $\bar{k}$, the upper bound on the cost of acquiring information. In particular, we know that as r increases, a larger fraction of parties will become informed in equilibrium, and we know from Conjecture 1 that the informativeness of the price system is increasing in the fraction of parties informed.

Conjecture 5 (GS 5) *When there is more noise, more players become informed in equilibrium.*

In our model, an increase in noise is interpreted as an increase in the variance of x, the noisy net supply available for our industry. The appendix shows that $\Delta(\lambda)$ is increasing in σ_x.

Conjecture 6 (GS 6) *"In the limit, when there is no noise, prices convey all information, and there is no incentive to purchase information. Hence, the only possible equilibrium is one with no information. But if everyone is uninformed, it clearly pays some individual to become informed. Thus, there does not exist a competitive equilibrium."*

This classic GS result–that equilibrium fails to exist when prices are perfectly informative–also holds in our model (as discussed in Theorem 2) and results from a discontinuity of the benefits of becoming informed, $\Delta(\lambda)$, at $\sigma_x = \lambda = 0$.

Conjecture 7 (GS 7) *Markets are thinner when λ^* is close to zero or one.*

As in GS, we define market thickness in state (x, v) as the per-capita trade (relative to net supply) of informed parties, or

$$\lambda\left(\frac{c_I(v, p_\lambda)}{\bar{c}} - x\right) = \lambda(1-\lambda)\frac{v - \mu_{v|p}}{\bar{c}}.$$

The expectation of the right-hand side is zero, and its variance goes to zero as λ tends to 0 or 1. This result is trivial in our model, since each party is limited to trading at most one widget, and as λ tends to 0, there are no informed parties for the uninformed parties to trade with (and correspondingly as λ tends to 1). Difficulties arise in GS, because as $\lambda \to 0$ in their model, the trade of informed parties could potentially explode at a fast enough rate to prevent the per-capita trade from tending to zero. Our model's capacity constraint on production ($q_i \in \{0, 1\}$) and hence trading prevents this from occurring.

4 Conclusion

Our goal was to provide a rational-expectations model of price formation with assumptions and features consistent with goods markets, as opposed to financial markets. We have developed and analyzed such a model, delivering simple and intuitive closed-form solutions for both the equilibrium price function and the equilibrium proportion of players who become informed. The appealing comparative statics of the Grossman-Stiglitz model also hold in our model.

Other authors have also analyzed rational-expectations equilibria with positive prices and quantities, as in goods markets, but these papers have focused on other issues and hence typically not delivered the closed-form price function and informed proportion that are important for both extensions and applications of the present paper. For example Ausubel (1990) exhibits a class of pure exchange economies in which partially revealing rational-expectations equilibria exist, are unique, and have non-negative prices, but his model does not typically produce a closed-form price function, and he takes which players are informed to be exogenous. As a second example, Barlevy and Veronesi (2000) consider a binomial distribution of the value of an asset (where the lowest value need not be negative) and an exponential shock to demand (so demands cannot be negative), but they focus on whether information can be a strategic complement among players[3]. Finally, Bai, Chang and Wang (2006) analyze a rational-expectations model with uniform random variables and risk-averse traders, but they focus on the impact of short-selling constraints on efficiency, again taking information acquisition decision as exogenous.

In Gibbons et al. (2009) we extend this model of price formation to a richer setting, where the single player in this model is replaced by a dyad of upstream and downstream parties. In a given dyad, the parties may or may not be integrated and they may or may not acquire information before participating in the intermediate-good market (where upstream parties may sell the intermediate good and downstream may buy). This model allows us to combine the single-dyad analysis of the integration decision that is typical of the transaction-cost and property-rights literatures with the present paper's analysis of the price mechanism in goods markets. In short, we analyze *markets* versus hierarchies.

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5 Appendix

5.1 Explicit Computations

Claim 1 $\frac{1}{\bar{x}-\underline{x}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\left[\sum_{j=1}^{3}\int_{R_j(v)}\left(v-\mu_{v|p}^{j}(x,v)\right)^2 dx\right]dv=\frac{\sigma_v^2}{2\bar{c}}\left(1-\frac{\lambda}{2}\frac{\sigma_v/\sigma_x}{\bar{c}}\right)$ *for case A.*

Proof. First, recall that in case A, we have

$$\begin{aligned}
\mu_{v|p}^{1}(x,v) &= \mu_v-\frac{\bar{v}-v}{2}+\frac{\frac{\bar{c}}{\lambda}(\bar{x}-x)}{2}\\
\mu_{v|p}^{2}(x,v) &= \mu_v\\
\mu_{v|p}^{3}(x,v) &= \mu_v+\frac{v-\underline{v}}{2}-\frac{\frac{\bar{c}}{\lambda}(x-\underline{x})}{2}.
\end{aligned}$$

Thus, we can write

$$\begin{aligned}
&\frac{1}{\bar{x}-\underline{x}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\left[\sum_{j=1}^{3}\int_{R_j(v)}\left(v-\mu_{v|p}^{j}(x,v)\right)^2 dx\right]dv\\
=\;&\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}(v-\mu_v)^2\,dxdv\\
&+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\int_{\bar{x}-\frac{\lambda}{\bar{c}}(\bar{v}-v)}^{\bar{x}}(v-\mu_v)\left(\bar{v}-v-\frac{\bar{c}}{\lambda}(\bar{x}-x)\right)dxdv\\
&-\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\underline{x}+\frac{\lambda}{\bar{c}}(v-\underline{v})}(v-\mu_v)\left(v-\underline{v}-\frac{\bar{c}}{\lambda}(x-\underline{x})\right)dxdv\\
&+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\int_{\bar{x}-\frac{\lambda}{\bar{c}}(\bar{v}-v)}^{\bar{x}}\left(\frac{\bar{v}-v}{2}-\frac{\frac{\bar{c}}{\lambda}(\bar{x}-x)}{2}\right)^2 dxdv\\
&+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\underline{x}+\frac{\lambda}{\bar{c}}(v-\underline{v})}\left(\frac{v-\underline{v}}{2}-\frac{\frac{\bar{c}}{\lambda}(x-\underline{x})}{2}\right)^2 dxdv.
\end{aligned}$$

By substitution and symmetry, this becomes

$$
\begin{aligned}
& \frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\left(v-\mu_v\right)^2 dxdv \\
& +2\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_0^{\bar{v}-\underline{v}}\int_0^{\frac{\lambda}{\bar{c}}u}\left(\bar{v}-u-\mu_v\right)\left(u-\frac{\bar{c}}{\lambda}w\right)dwdu \\
& +2\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\frac{1}{2\bar{c}}\int_0^{\bar{v}-\underline{v}}\int_0^{\frac{\lambda}{\bar{c}}u}\left(\frac{u}{2}-\frac{\frac{\bar{c}}{\lambda}w}{2}\right)^2 dwdu \\
= & \frac{\sigma_v^2}{2\bar{c}}-\frac{1}{2\bar{c}}\frac{\lambda\sigma_v}{\bar{c}\sigma_x}\sigma_v^2-\frac{1}{2\bar{c}}\frac{1}{2}\frac{\lambda\sigma_v}{\bar{c}\sigma_x}\sigma_v^2=\frac{\sigma_v^2}{2\bar{c}}\left(1-\frac{\lambda}{2}\frac{\sigma_v/\sigma_x}{\bar{c}}\right),
\end{aligned}
$$

establishing the claim. ■

Claim 2 *$E_{x,v}\left[\sigma^2_{v|p}\right]=\left(\frac{\bar{c}}{\lambda}\right)^2\sigma_x^2\left(1-\frac{1}{2}\frac{1}{\lambda}\frac{\bar{c}}{\sigma_v/\sigma_x}\right)$ for case B.*

Proof. Omitted but established similarly for case B as for case A. ■

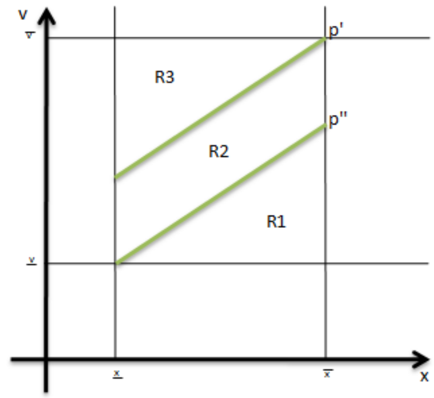
Claim 3 *$\frac{\partial\Delta(\lambda)}{\partial\sigma_v}>0$ and $\frac{\partial\Delta(\lambda)}{\partial\sigma_x}>0$ for all λ and the sign of $\frac{\partial\Delta(\lambda)}{\partial\bar{c}}$ is indeterminate.*

Proof. Omitted. ■

5.2 Case B Analysis

For now, let us treat λ as exogenous. Assume that λ is such that we are in case B. Define

$$
\begin{aligned}
R_\lambda^1 &= \left\{(x,v):p_\lambda(x,v)\le\underline{p}\right\} \\
R_\lambda^2 &= \left\{(x,v):\underline{p}<p_\lambda(x,v)\le\bar{p}\right\} \\
R_\lambda^3 &= \left\{(x,v):\bar{p}<p_\lambda(x,y)\right\}.
\end{aligned}
$$



Case B Regions

Region 1

If $p < \underline{p}$, then individuals believe that $v|\, p_\lambda(\cdot,\cdot) = p \sim U\left[\underline{v}^1(p), \bar{v}(p)\right]$, where $\bar{v}^1(p) = \frac{1}{\beta_1}\left(p - \beta_0^1 + \beta_2^1 \bar{x}\right)$, so that

$$\mu^1_{v|p} = \frac{\underline{v}^1(p) + \bar{v}^1(p)}{2} = \frac{\underline{v} + \left(p - \beta_0^1 + \beta_2^1 \bar{x}\right)/\beta_1^1}{2}.$$

Market-clearing ensures that

$$\frac{1}{2}\left[\underline{v} + \left(\underbrace{\left(\beta_0^1 + \beta_1^1 v - \beta_2^1 x\right)}_{p^1_\lambda(x,v)} - \beta_0^1 + \beta_2^1 \bar{x}\right)/\beta_1^1\right] = \frac{1}{1-\lambda}\left(\underbrace{\left(\beta_0^1 + \beta_1^1 v - \beta_2^1 x\right)}_{p^1_\lambda(x,v)} + \bar{c}x - \lambda v\right).$$

If we solve for β_0^1, β_1^1, and β_2^1 noting that this holds as an identity we obtain

$$\begin{aligned}
\beta_0^1 &= (1-\lambda)\frac{\underline{v} + (\bar{c}/\lambda)\,\bar{x}}{2} \\
\beta_1^1 &= \frac{1+\lambda}{2} \\
\beta_2^1 &= \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}.
\end{aligned}$$

Region 2

If $\underline{p} < p < \bar{p}$, then individuals believe that $v|\, p_\lambda(\cdot,\cdot) = p \sim U\left[\underline{v}^2(p), \bar{v}^2(p)\right]$, where $\bar{v}^2(p) = \frac{1}{\beta_1}\left(p - \beta_0^1 + \beta_2^1 \bar{x}\right)$ and $\underline{v}^2(p) = \frac{1}{\beta_1}\left(p - \beta_0^3 + \beta_2^3 \underline{x}\right)$, so that

$$\begin{aligned} \mu^2_{v|p} &= \frac{\underline{v}^2(p) + \bar{v}^2(p)}{2} \\ &= \frac{\left(p - \beta_0^1 + \beta_2^1 \bar{x}\right)/\beta_1^2 + \left(p_\lambda^2(x,v) - \beta_0^3 + \beta_2^3 \underline{x}\right)/\beta_1^2}{2}. \end{aligned}$$

Market-clearing ensures that

$$\begin{aligned} &\frac{1}{2}\left[\left(\underbrace{\beta_0^2 + \beta_1^2 v - \beta_2^2 x}_{p_\lambda^2(x,v)} - \beta_0^1 + \beta_2^1 \bar{x}\right)/\beta_1^2 + \left(\underbrace{\beta_0^2 + \beta_1^2 v - \beta_2^2 x}_{p_\lambda^2(x,v)} - \beta_0^3 + \beta_2^3 \underline{x}\right)/\beta_1^2\right] \\ = \;& \frac{1}{1-\lambda}\left(\underbrace{\beta_0^2 + \beta_1^2 v - \beta_2^2 x}_{p_\lambda^2(x,v)} + \bar{c}x - \lambda v\right) \end{aligned}$$

Solving for β_0^2, β_1^2, and β_2^2 yields

$$\begin{aligned} \beta_0^2 &= (1-\lambda)\frac{\bar{c}}{\lambda}\frac{\underline{x} + \bar{x}}{2}, \\ \beta_1^2 &= 1 \\ \beta_2^2 &= \frac{\bar{c}}{\lambda}. \end{aligned}$$

Region 3

Finally, if $\bar{p} < p$, then individuals believe that $v|\, p_\lambda(\cdot,\cdot) = p \sim U\left[\underline{v}^3(p), \bar{v}^3(p)\right]$, where $\underline{v}^3(p) = \left(p - \beta_0^3 + \beta_2^3 \underline{x}\right)/\beta_1$, so that

$$\begin{aligned} \mu^3_{v|p} &= \frac{\underline{v}^3(p) + \bar{v}^3(p)}{2} \\ &= \frac{\frac{1}{\beta_1}\left(p - \beta_0^3 + \beta_2^3 \underline{x}\right) + \bar{v}}{2}. \end{aligned}$$

Market-clearing ensures that

$$\frac{1}{2}\left[\left(\underbrace{\beta_0^3+\beta_1^3 v-\beta_2^3 x}_{p_\lambda^3(x,v)}-\beta_0^3+\beta_2^3\underline{x}\right)/\beta_1^3+\bar{v}\right]=\frac{1}{1-\lambda}\left(\underbrace{\beta_0^3+\beta_1^3 v-\beta_2^3 x}_{p_\lambda^3(x,v)}+\bar{c}x-\lambda v\right)$$

Solving as above yields

$$\begin{aligned}
\beta_0^3 &= (1-\lambda)\frac{(\bar{c}/\lambda)\,\underline{x}+\bar{v}}{2}\\
\beta_1^3 &= \frac{1+\lambda}{2}\\
\beta_2^3 &= \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}.
\end{aligned}$$

To summarize, we have the following

$$\begin{aligned}
p_\lambda^1(x,v) &= (1-\lambda)\frac{\underline{v}+(\bar{c}/\lambda)\,\bar{x}}{2}+\frac{1+\lambda}{2}v-\frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x\\
p_\lambda^2(x,v) &= (1-\lambda)\frac{\bar{c}}{\lambda}\frac{\underline{x}+\bar{x}}{2}+v-\frac{\bar{c}}{\lambda}x\\
p_\lambda^3(x,v) &= (1-\lambda)\frac{(\bar{c}/\lambda)\,\underline{x}+\bar{v}}{2}+\frac{1+\lambda}{2}v-\frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}x
\end{aligned}$$

By construction, our conjectured equilibrium pricing function

$$\begin{aligned}
p_\lambda(x,v) &= 1_{\left\{(x,v)\in R_\lambda^1\right\}}\left[\beta_0^1+\beta_1^1 v-\beta_2^1 x\right]+1_{\left\{(x,v)\in R_\lambda^2\right\}}\left[\beta_0^2+\beta_1^2 v-\beta_2^2 x\right]\\
&\quad +1_{\left\{(x,v)\in R_\lambda^3\right\}}\left[\beta_0^3+\beta_1^3 v-\beta_2^3 x\right]\\
&\equiv 1_{\left\{(x,v)\in R_\lambda^1\right\}}p_\lambda^1(x,v)+1_{\left\{(x,v)\in R_\lambda^2\right\}}p_\lambda^3(x,v)+1_{\left\{(x,v)\in R_\lambda^3\right\}}p_\lambda^3(x,v)
\end{aligned}$$

is a fixed point of the equation

$$E\left[v|\,p_\lambda(\cdot,\cdot)=p_\lambda(x,v)\right]=\frac{1}{1-\lambda}\left(p_\lambda(x,v)+\bar{c}x-\lambda v\right),$$

where

$$\mu_{v|p}(x,v) = \begin{cases} \left(v - \frac{\bar{c}}{\lambda}x\right) + \frac{\bar{c}}{\lambda}\frac{\bar{x}+\underline{x}}{2} - \frac{1}{2}(v - \underline{v}) + \frac{1}{2}\frac{\bar{c}}{\lambda}(x - \underline{x}) & \text{if} \quad p_\lambda(x,v) \le \underline{p} \\ \left(v - \frac{\bar{c}}{\lambda}x\right) + \frac{\bar{c}}{\lambda}\frac{\bar{x}+\underline{x}}{2} & \text{if} \quad \underline{p} < p_\lambda(x,v) \le \bar{p} \\ \left(v - \frac{\bar{c}}{\lambda}x\right) + \frac{\bar{c}}{\lambda}\frac{\bar{x}+\underline{x}}{2} + \frac{1}{2}(\bar{v} - v) - \frac{1}{2}\frac{\bar{c}}{\lambda}(\bar{x} - x) & \text{if} \quad \bar{p} < p_\lambda(x,v). \end{cases}$$

Let us verify that we are indeed in case B–i.e. Is it the case that $\bar{p} > \underline{p}$?

$$\bar{p} = (1-\lambda)\frac{\underline{v} + (\bar{c}/\lambda)\,\bar{x}}{2} + \frac{1+\lambda}{2}\bar{v} - \frac{1+\lambda}{2}\frac{\bar{c}}{\lambda}\bar{x} > (1-\lambda)\frac{\bar{c}}{\lambda}\frac{\underline{x}+\bar{x}}{2} + \underline{v} - \frac{\bar{c}}{\lambda}\underline{x} = \underline{p}.$$

This holds if

$$\lambda > \bar{c}\frac{\sigma_x}{\sigma_v}$$

Thus, there is a nontrivial region of the endogenous parameter space for which this is a valid fixed point. Note that, unlike case A, it is *not* possible to ensure that we are always in case B by making reasonable exogenous parameter restrictions.