For next week:

Josh's SR demand for Palestinian Labor

Card, Borjas, Cortes } David Autor will probably cover this

Card-Kreuger (1994)

- write down all the problems you see with it.

2SLS bias

$$\hat{\beta}_{2SLS} = (X' P_Z X)^{-1} X' P_Z Y = (\hat{X}'\hat{X})^{-1} \hat{X}' Y$$

$$= (X' P_Z X)^{-1} X' P_Z (X\beta + \varepsilon)$$

$$= \beta + (X' P_Z X)^{-1} X' P_Z \varepsilon$$

$$\to \beta + (\Sigma_{XZ} \Sigma_{ZZ}^{-1} \Sigma_{ZX})^{-1} \Sigma_{XZ} \Sigma_{ZZ} \underbrace{\Sigma_{Z\varepsilon}}_{=0} = \beta$$

$\hat{\beta}_{2SLS}$ is consistent

$$y_1 = y_2 \beta + \varepsilon$$

$$y_2 = Z\pi + v$$

second-order Taylor approximation

$$E[\hat{\beta}_{2SLS}] - \beta \approx \frac{K\rho(1-R^2)}{nR^2}$$

- increases w/ degree of overid
- decreases in n
- decreases in R^2

If just id:

$$\hat{\beta}_{2SLS} = (X'Z(Z'Z)^{-1}\overbrace{Z'X}^{\text{square}})^{-1} X'Z(Z'Z)^{-1}Z'Y$$

$$= (Z'X)^{-1} Z'Z(X'Z)^{-1} X'Z(Z'Z)^{-1}Z'Y$$

$$= (Z'X)^{-1}Z'Y \quad \text{has no moments}$$

$E[\hat{\beta}_{2SLS}] - \beta$ does not exist.

Production side

$$Y = F(K,L) \qquad (F(\alpha K, \alpha L) = \alpha F(K,L)), \; F_{KL} > 0$$

$F_K > 0$
$F_L > 0$
$F_{LL} < 0$
$F_{KK} < 0$

$$\max_{K,L} \; F(K,L) - wL - rK$$

FOCs: $(K): F_K = r$

$(L): F_L = w$

Let $\sigma \equiv \left.\dfrac{\partial \log(K/L)}{\partial \log(w/r)}\right|_Y$ elasticity of substitution

$$= \left.\frac{\partial \log(K/L)}{\partial \log(F_K/F_L)}\right|_Y$$

Proposition: $\sigma = \dfrac{F_L F_K}{Y F_{LK}} > 0$

Proof:

$$d\left(\frac{F_L}{F_K}\right) = \frac{\partial(F_L/F_K)}{\partial K}dK + \frac{\partial(F_L/F_K)}{\partial L}dL$$

$$0 = F_L dL + F_K dK$$

$$\Rightarrow \quad dL = -\frac{F_K}{F_L}dK \qquad (*)$$

$$\Rightarrow d\left(\frac{F_L}{F_K}\right) = \frac{\partial(F_L/F_K)}{\partial K}dK - \frac{\partial(F_L/F_K)}{\partial L}\frac{F_K}{F_L}dK$$

$$= \left[F_L\frac{\partial(F_L/F_K)}{\partial K} - F_K\frac{\partial(F_L/F_K)}{\partial L}\right]\frac{dK}{F_L}$$

Next: $$d\left(\frac{K}{L}\right) = \frac{\partial(K/L)}{\partial K}dK + \frac{\partial(K/L)}{\partial L}dL$$

$$= \frac{1}{L}dK - \frac{K}{L^2}dL$$

plugging in (*)

$$= \frac{LdK - KdL}{L^2} = \left(LF_L + KF_K\right)\frac{dK}{F_L L^2}$$

$$\sigma = \frac{d(K/L)}{d(F_L/F_K)} \cdot \frac{F_L/F_K}{K/L}$$

$$= \frac{\left(LF_L + KF_K\right)\frac{dK}{F_L L^2}}{\left(F_L\frac{\partial F_L/F_K}{\partial K} - F_K\frac{\partial(F_L/F_K)}{\partial L}\right)\frac{dK}{F_L}} \cdot \frac{F_L/F_K}{K/L}$$

$$= \frac{(LF_L + KF_K)\frac{dK}{F_L L^2}}{\left(F_L \frac{F_K F_{LK} - F_L F_{KK}}{F_K^2} - F_K \frac{F_{LL} F_K - F_L F_{LK}}{F_K^2}\right)\frac{dK}{F_L}} \cdot \frac{F_L/F_K}{K/L}$$

$$= \frac{(LF_L + KF_K) F_K F_L}{LK(2F_L F_K F_{LK} - F_L^2 F_{KK} - F_K^2 F_{LL})}$$

Recall: $F = F_L L + F_K K$ (Euler's Theorem)

$$F_L = F_L + LF_{LL} + KF_{KL}$$

$$\Rightarrow F_{LL} = -\frac{K}{L} F_{KL}$$

and $F_{KK} = -\frac{L}{K} F_{KL}$

Thus,

$$\sigma = \frac{Y F_K F_L}{LK\left(2F_L F_K F_{LK} + F_L^2 \frac{L}{K} F_{LK} + F_K^2\left(\frac{K}{L} F_{LK}\right)\right)} = \frac{Y F_K F_L}{Y^2 F_{KL}} = \frac{F_K F_L}{Y F_{KL}}$$

Define $\eta_{Lw} = \left.\frac{d\log L}{d\log w}\right|_Y$, $\eta_{Lr} = \left.\frac{d\log L}{d\log r}\right|_Y$

<u>Prop 2:</u> $\eta_{Lw} = -(1-s)\sigma < 0$ where $s = \frac{wL}{Y}$

<u>Prop 3:</u> $\eta_{Lr} = (1-s)\sigma > 0$

Now, allowing output to change

$\eta_{Lw}' = -(1-s)\sigma - s\eta$, where $\eta \equiv \dfrac{d\log Y}{d\log p}$

= Dual problem:

$\min_{L,K} \; wL + rK \quad \text{s.t.} \quad F(L,K) = Y$

$$C = C(w,r,Y) = wL^*(w,r,Y) + rK^*(w,r,Y)$$

Hotelling's Lemma: $C_w = L^*$

Prop: $\sigma = \dfrac{C\,C_{wr}}{C_w C_r}$