$$U_t \equiv \sum_{s=t}^{T} \beta^{s-t} E_t[u(C_s, \underbrace{H_s}_{\text{hours worked}})]$$

$$\cancel{W_t \equiv A_{t-1}(1+r) + E_t\left[\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t}(w_s H_s - C_s)\right]}$$

$$W_t = (1+r)[W_{t-1} + w_{t-1}H_{t-1} - C_{t-1}]$$

$$V(W_t) = \max_{C_t, H_t, \lambda_t, W_{t+1}} \left\{ u(C_t, H_t) + \beta E_t[V(W_{t+1})] + \lambda_t\left[W_t + w_t H_t - C_t - W_{t+1}\frac{1}{1+r_t}\right]\right\}$$

FOCs:

(C_t): $U_C(C_t, H_t) = \lambda_t \Rightarrow C_t^* = C(\lambda_t, w_t)$

(H_t): $U_H(C_t, H_t) = -\lambda_t w_t \Rightarrow H_t^* = H(\lambda_t, w_t)$

Frisch demand holds λ_t constant

(W_{t+1}): $(1+r_t)\beta E_t[V'(W_{t+1})] = \lambda_t \Rightarrow$

EC:

(W_t): $V'(W_t) = \lambda_t$

$$\Rightarrow (1+r_t)\beta E_t[\lambda_{t+1}] = \lambda_t$$

$$\log H_{it} = A_i + \eta \log w_{it} + \Gamma \log \lambda_{it} + q_{it}$$

1] cross section (let λ_{it} be unobserved)
- can't do this for obvious reasons.

2] $\Delta \log H_{it} = \eta \, \Delta \log(w_{it}) + \gamma [\log(\lambda_{it}) - \log(\lambda_{it-1})] + \Delta \varepsilon_{it}$

$\Rightarrow \Delta \log H_{it} = \eta \Delta \log(w_{it}) + \gamma [\underbrace{\log(\lambda_{it}) - \log(E_{t-1}(\lambda_{it}))}_{\text{forecasting error}}] + k + \Delta \varepsilon_{it}$

- forecasting error will be correlated with w_{it}.
  - ie both could be caused by "losing an arm."

3] Groups:

$$\overline{\Delta \log H_{it}} = \eta \, \overline{\Delta \log w_{it}} + \gamma [\underbrace{\overline{\log \lambda_{it}} - \overline{\log E_{t-1}(\lambda_{it})}}_{\text{assume } = 0.}] + k + \overline{\Delta \varepsilon_{it}}$$

$$\hat{\eta} = \frac{\overline{\log H_{it}} - \overline{\log H_{it-1}}}{\overline{\log w_{it}} - \overline{\log w_{it-1}}}$$

2 groups (years)
- do we get the same $\hat{\eta}$ for different sets of groups?

$$\hat{\beta}_{2SLS} = (\bar{X}'\bar{X})^{-1}\bar{X}'\bar{Y} = \frac{\sum_{k=1}^{K} R \bar{X}_k (\bar{Y}_k - \bar{\bar{Y}})}{\sum_{k=1}^{K} R \bar{X}_k (\bar{X}_k - \bar{\bar{X}})}$$

K groups

$$= \frac{\sum_{k=1}^{K} R \bar{X}_k \sum_{l=1}^{K} \frac{R}{N} (\bar{Y}_k - \bar{Y}_l)}{}$$

$$= \frac{\sum_{k=1}^{K} \sum_{l>k} R^2 (\bar{X}_k - \bar{X}_l)(\bar{Y}_k - \bar{Y}_l)}{}$$

$$= \frac{\sum_{k=1}^{K} \sum_{l>k} R^2 (\bar{\bar{X}}_k - \bar{\bar{X}}_l)^2 \frac{\bar{\bar{Y}}_k - \bar{\bar{Y}}_l}{\bar{\bar{X}}_k - \bar{\bar{X}}_l}}{\sum_{k=1}^{K} R \bar{\bar{X}}_k (\bar{\bar{X}}_k - \bar{\bar{X}})}$$

$$= \sum_{k=1}^{K} \sum_{l>k} \left[\underbrace{\frac{R^2 (\bar{\bar{X}}_k - \bar{\bar{X}}_l)^2}{R \bar{\bar{X}}_k (\bar{\bar{X}}_k - \bar{\bar{X}})}}_{\text{weights based on heteroskedasticity.}} \cdot \underbrace{\left(\frac{\bar{\bar{Y}}_k - \bar{\bar{Y}}_l}{\bar{\bar{X}}_k - \bar{\bar{X}}_l} \right)}_{\text{pairwise Wald estimates}} \right]$$