Intertemporal movements (for individuals "age effects")
aggregate components ("year effects")
Cross-sectional components ("person effects" if time invariant)

any within-person variance over time other than age effects (some of this might be intertemporal substitution)
Cohort effects: cohort crowding is a possible cause
The hours profile is much flatter than the wage profile.
Educated workers work more, but this doesn't make sense, since they make more forever and hence wealth effects would predict that hours would be lower.

Perfect information

$$\max \; u(c_{i1}, \ldots, c_{iT}; l_{i1}, \ldots, l_{iT})$$

- $w_{i1}, \ldots, w_{iT}$
- $p_1, \ldots, p_T$ (invariant across people)
- A_0

$$\text{s.t. } A_0 = \sum_{t=1}^{T} \frac{p_t c_{it} - w_{it}(T - l_{it})}{1+rt}$$

First simplification: intertemporal additivity

$$U = \sum_{t=0}^{T} u_t (c_{it}, l_{it})$$

Second simplification: constant discounting and stable w/in period utility fcn:

$$U = \sum_{t=0}^{T} \frac{u(c_{it}, l_{it})}{(1+\rho)^t}$$ Benchmark life-cycle model

- There is a fair amount of functional form inherent in this additivity
- There are a lot of assumptions in the budget constraint
  - perfect credit markets
  - parametric prices
- adding uncertainty changes this model surprisingly little.

$$\max_{\{c_{it}\}, \{l_{it}\}} \sum_{t=0}^{T} \frac{u(c_{it}, l_{it})}{(1+\rho)^t}$$

$$\text{s.t. } A_0 = \sum_{t=0}^{T} \frac{P_t (c_{it} - w_{it}(T - l_{it}))}{(1+r)^t}$$

$$h = \sum_{t=0}^{T} \left(\frac{1}{1+\rho}\right)^t u(c_{it}, l_{it}) - \lambda_i \left[A_0 - \sum_{t=0}^{T} \frac{P_t c_{it} - w_{it}(T - l_{it})}{(1+r)^t}\right]$$

FOCs:

$$(c_{it}): \left(\frac{1}{1+\rho}\right)^t u_c(c_{it}, l_{it}) = \lambda_i P_t \frac{1}{(1+\rho)^t} \qquad t = 0, \dots, T$$

$$(l_{it}): \left(\frac{1}{1+\rho}\right)^t u_l(c_{it}, l_{it}) = \lambda_i \frac{w_{it}}{(1+r)^t} \qquad t = 0, \dots, T$$

$$\Rightarrow \quad c_{it} = f(r, \rho, \lambda_i, P_t, w_{it})$$

$$l_{it} = g(r, \rho, \lambda_i, P_t, w_{it})$$

- plug this into budget constraint, solve for λ_i
- We can typically stop here.
  - λ_i has everything we need to know about all future wages and prices.

    $\underbrace{\lambda_i}_{\text{marginal utility of wealth}}$ = complicated f^n $(w_{i0}, \dots, w_{iT}, P_0, \dots, P_T, A_0)$
- can introduce $w_{it}(l_{it-1}, \dots, l_{i0})$, but we will not now.

Lucas-Rapping (1970), Heckman-Macurdy (1980)

Let $u(c_{it}, h_{it}) = \eta_{1i} c_{it}^{\delta_1} - \eta_{2i} h_{it}^{\delta_2}$

- within period additivity.

FOC:

$$(h_{it}) \quad \frac{\delta_2 \eta_{2i} h_{it}^{\delta_2 - 1}}{(1+\rho)^t} = \lambda_i \frac{w_{it}}{(1+r)^t}$$

$$\Rightarrow \eta_{2i} \delta_2 h_{it}^{\delta_2 - 1} = \left(\frac{1+\rho}{1+r}\right)^t \lambda_i w_{it}$$

$\ln\left(\frac{1+\rho}{1+r}\right)^t = t[\ln(1+\rho) - \ln(1+r)] \approx t(\rho - r)$ since $\ln(1+\rho) \approx \rho$

$$\Rightarrow \log \eta_{2i} + \log \delta_2 + (\delta_2 - 1)\log h_{it} = t(\rho - r) + \ln \lambda_i + \ln w_{it}$$

$$\Rightarrow \log h_{it} = \underbrace{(\cdot)}_{\text{everything here doesn't depend on } t \text{. person effect}} + \frac{1}{\delta_2 - 1}\ln w_{it} + \underbrace{t(\rho - r)}_{\text{trend}}$$

- wages affect contemporaneous labor supply $\frac{1}{\delta_2 - 1}$

$$\left.\frac{\partial \log h_{it}}{\partial \log w_{it}}\right|_{\lambda_i \text{ fixed}} = \frac{1}{\delta_2 - 1} \equiv \underbrace{\delta}_{\text{Intertemporal labor supply elasticity}}$$

(need $\delta_2 > 1$ for SOCs to hold)

- Heckman-Macurdy and Stone-Geary preferences are the ones most often used

Why is δ important? It answers the question: "What are the consequences of a change in wages that don't affect lifetime wealth."