Exam begins at 8:30 on Thursday.
<u>another</u> practice exam will be posted.

- Consumption
- asset prices
- complete/incomplete markets.

} until now

Now, investment with idiosyncratic shocks?

- Frictions to investment
- Interest rate is typically the marginal product depreciation (physical) and economic depreciation
- Capital usually adjusts immediately
- What if we have convex adjustment costs? ie easier to adjust capital slowly
  - This gave us the Q-model of investment
    - difference between replacement price and price of installed capital since costly installation.
  - This predicts a smoother adjustment process
- at the micro level investment is usually zero (plant level). Cluster of large investments.
- Once you install capital, it is hard to disinstall (irreversibility.)
- Can think of this as a more interesting theory of labor demand (irreversibility $\Leftrightarrow$ firing costs)
  - Davis and Haltiwanger ('94) - employment evidence
  - Chap 2 - Caballero book - investment evidence

User cost

Value w/ complete markets

$$\sum_{t,s^t} q_t^0(s^t)\underbrace{d_t(s^t)}_{\text{net dividends}}$$

$$d_t(s^t) = \overbrace{\pi(k_t(s^{t-1}), s_t) - \underbrace{c(k_t(s^{t-1}), i_t(s^t), s_t)}_{\text{cost of adjusting capital: } c_{ii}>0}}^{B(s^t)} - \overbrace{p_t(s^t)}^{\text{price of machine}} i_t(s^t)$$

$$k_{t+1}(s^t) = (1-\delta)k_t(s^{t-1}) + i_t(s^t)$$

$$\Rightarrow d_t(s^t) = B_t(s^t) - p_t(k_{t+1}(s^t) - (1-\delta)k_t(s^{t-1}))$$

$\Rightarrow$

$$\text{Value: } \sum_{t,s^t} q_t^0(s^t)\left[B_t(s^t) - \underbrace{p_t(k_{t+1}(s^t) - (1-\delta)k_t(s^{t-1}))}_{\text{user cost: } v(s^t)}\right]$$

- rental rate

If $p_{t+1}(s^{t+1}) = p_t(s^t)$, then

$$q_t^0(s^t)k_{t+1}(s^t) - \sum_{s_{t+1}} q_{t+1}^0(s^{t+1})(1-\delta)k_{t+1}(s^t)$$

$$= q_t^0(s^t)k_{t+1}(s^t)\left(1 - \frac{1-\delta}{R_{t,t+1}(f)}\right), \quad R(s^t) = \frac{q_t^0(s^t)}{\sum_{s_{t+1}} q_{t+1}^0(s^{t+1})}$$

$$= \sum_{s_{t+1}} q_t^0(s^t)k_{t+1}(s^t)(R(s^t) - (1-\delta))$$

$$\Rightarrow v(s^t) = p_t(s^t)(R(s^t) - (1-\delta))$$

If $p_t(s^t) \neq p_{t+1}(s^t, s_{t+1}) = p_{t+1}(s^t, \tilde{s}_{t+1})$ (non-constant but predictable)

$$v(s^t) = p_t(s^t)q_t^0(s^t)k_{t+1}(s^t) - \sum_{s_{t+1}} p_{t+1}(s^{t+1})q_{t+1}^0(s^{t+1})(1-\delta)k_{t+1}(s^t)$$

$$= p_t(s^t)\left(R(s^t) - (1-\delta)\underbrace{\frac{p_{t+1}(s^{t+1})}{p_t(s^t)}}_{\text{economic depreciation}}\right)$$

Economic depreciation: $(1-\delta_\varepsilon) = (1 - \frac{p_{t+1}(s^{t+1})}{p_t(s^t)}) = (\frac{p_t(s^t) - p_{t+1}(s^{t+1})}{p_t(s^t)})$

$$\Rightarrow \delta^\varepsilon = (1 - \frac{\Delta p_{t,t+1}(s^{t+1})}{p_t(s^t)})$$

$$\Rightarrow \nu(s^t) = p_t(s^t)\left(R(s^t) - (1-\delta)(1-\delta_\varepsilon)\right)$$

Thus,

$$\underline{\tilde{d}_t(s^t) = \pi(k_t(s^{t-1}), s_t) - c(k_t(s^{t-1}), i_t(s^t), s_t) - \nu(s_t)k_t(s^{t+1})}$$

Irreversibility:

Three periods: $t \in \{-1, 0, 1\}$

$t=-1$: invest k_0

$t=0$: invest k_1

at $t=0$: learn $A_1 \in \{A_H, A_L\}$ $\Pr[A_H] = p$

irreversibility: $k_1 \geq k_0(1-\delta)$ (cannot have less than depreciated capital - cannot sell capital.)

RN pricing: $q_t^0(s^t) = R^{-t}\Pr[s^t]$

$\cdot\, \nu(s_t) = \nu \quad \forall s_t.$

Firm: known productivity

$$\max_{k_0} \{\underset{\sim}{A_0} F(k_0) - \nu k_0 + R^{-1}[p \cdot \max_{k_1 \geq k_0(1-\delta)} (A^H F(k_1) - \nu k_1) + (1-p) \cdot \max_{k_1 \geq k_0(1-\delta)} (A^L F(k_1) - \nu k_1)]$$

Equivalently:

$$\max_{k_0} A_0 F(k_0) - \nu k_0 + R^{-1} E[V(k_0, A_1)]$$

where $V(k_0, A) = \max_{k_1 \geq k_0(1-\delta)} A F(k_1) - \nu k_1$

Unconstrained:

$$\max_{k'} \; AF(k') - \nu k' \Rightarrow AF'(\overbrace{k^*(A)}^{\text{unconstrained optimum}}) - \nu = 0$$

$$k' = \max\{(1-\delta)k, k^*\}$$

If $k_1^*(A_L) = k_0(1-\delta)$

$$\Rightarrow AF'(k_1) - \nu \le 0$$

$$\Rightarrow A_0 F'(k_0) - \nu + \frac{1-p}{R}\overbrace{(A_L F'(k_1) - \nu)}^{(-)}(1-\delta) = 0$$

$$\Rightarrow k_0^* < k_0^*(A_0)$$

$$\Leftrightarrow A_0 F'(k_0) - \nu + \underbrace{E[V_k(k_0, A_1)]}_{<0} = 0$$

- Less investment in the face of irreversibility.
  - Lower capital stock in first period
  - More capital than I wanted in second period, on average.

More uncertainty $\Rightarrow$ more cautious

Investment rule:

$(1-\delta)k_0 \ge k_1^* \rightarrow$ don't invest

$(1-\delta)k_0 < k_1^* \rightarrow$ invest until $k_1 = k_1^*$

Bentolila and Bertola (Labor Demand)

$$\pi(k,z) = k^\alpha z^{1-\alpha}$$

Bellman:

$$V(k_-, z_-) = E\left[\max_{k \ge (1-\delta)k_-} \left[\pi(k,z) - rk + \overset{R^{-1}}{\beta} V(k,z)\right]\right]$$

assume log z is a random walk

Guess $V(k,z) = zV\left(\frac{k}{z},1\right) = zv\left(\frac{k}{z}\right)$

$$\Rightarrow z_- V\left(\frac{k_-}{z_-},1\right) = E\left[\max_{\frac{k}{z} \geq (1-\delta)\frac{z_-}{z}\frac{k_-}{z_-}} \left\{ z\left(\pi\left(\frac{k}{z},1\right) - r\frac{k}{z}\right)\right) + \beta z V\left(\frac{k}{z},1\right)\right]$$

Let $\varepsilon = \frac{z}{z_-}$

$$\Rightarrow v(k_-) = E\left[\varepsilon \max_{k \geq \varepsilon^{-1}(1-\delta)k_-} \left(\pi(k) - rk + \beta v(k)\right)\right]$$

unconstrained: $\pi'(k^*) = r - \beta V'(k^*)$

∘ Still a barrier control problem.

Can also put fixed costs in adjustment.