No commitment [Ligon/Thomas-Worral - hidden savings]
Thomas-Worral/Kocherlakota/Alvarez-Jermann

- Endowment shocks $s \mapsto y(s)$
- friction: agents can leave contract
- Infinite horizon. "reputational equilibrium", Markov structure

$$V_t(s^t) = \text{continuation utility} = E_t\left[\sum_{s=0}^{\infty} \beta^s u(c_{t+s})\right]$$

- outside option is autarky
  - $V_{AUT}(s) = E\left[\sum_{s=0}^{\infty} \beta^s u(y_s) \mid s_0 = s\right]$
  - assuming agents cannot store
  - more generally, can think of any $V_{AUT}(s)$
    - could result from savings technology after leaving town
    - needs to be history independent.

$$K(v,s) = \min_{c,v'} \{ \underbrace{c}_{\text{today's cost}} + \underbrace{q E[K(v'(s'),s') \mid s]}_{\text{discounted, expected future cost}} \}$$

$$u(c) + \beta E[v'(s) \mid s] = v \qquad (IR) \quad (\lambda)$$

- if there is an aggregate savings technology, we will have $q = R^{-1}$

$$\pi(s'|s) \underbrace{v'(s')}_{\text{promise for tomorrow}} \geq V_{aut}(s') \quad \forall s' \qquad (IC) \quad (\mu(s'))$$

Domain restriction: $\underbrace{v}_{\text{promise for today from yesterday}} \geq \bar{V}_{aut}(s)$

$K(\cdot, s)$ will be increasing, convex, differentiable.

$\Rightarrow$ FOCs necessary and sufficient.

(c): $\lambda u'(c) = 1$

$(v'(s'))$: $\underbrace{q K_v(v'(s'), s')}_{\text{MC of increasing } v'(s')} = \underbrace{\beta \lambda + \mu(s')}_{\text{two benefits of increasing } v'(s')}$

Policy functions: ◦ $g^c(v, s)$ $\quad (\mu(s') > 0 \Rightarrow v'(s') = \bar{v}_{aut}(s'))$

◦ $g^{v'}(s' | v, s)$

◦ use (c) to get $c(\lambda) = (u')^{-1}(1/\lambda)$

◦ use $(v'(s'))$: $q K_v(v_u'(s'), s') = \beta \lambda$

$\forall s'$ s.t. $v_u'(s') \geq \bar{v}_{aut}(s') \Rightarrow v'(s') = v_u'(s')$

otherwise $v'(s') = \bar{v}_{aut}(s')$.

◦ for a given v, s, must pick the right λ.

<u>EC</u>: $K_v(v, s) = \lambda$

$\Rightarrow g^c(v, s) = \tilde{g}^c(\lambda) = \tilde{g}^c(K_v(v, s)) \Rightarrow \tilde{g}^c = (u')^{-1}(\cdot)$

$g^v(s' | v, s) = \hat{g}^v(s' | \lambda) = \hat{g}^v(s' | K_v(v, s))$

Focus on λ and see how it evolves:

(EC): $\lambda'(s') = K_v(v'(s'), s') = K_v(\hat{g}^v(s' | \lambda), s')$

$(2) \Rightarrow q \lambda'(s') = \beta \lambda + \mu(s')$

In the FB world, λ doesn't move around

Can think of λ as Pareto weights for consumers. If we ignore $\mu(s')$ for now, then λ evolves only due to differences in discount factors:

$$\lambda_u'(s') = \frac{\beta}{q}\lambda$$

if constraint is not binding

When the IC binds, we have:

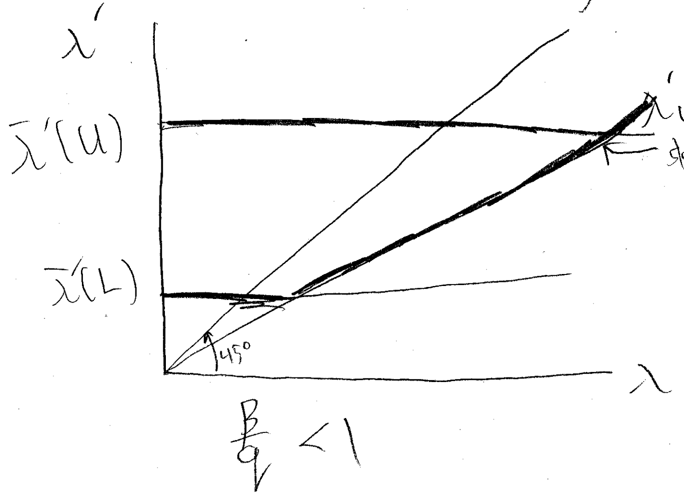
$$\bar{\lambda}'(s') = K_v(\bar{V}_{aut}(s'), s')$$

i.e. we will need to increase the individual's pareto weight to keep him in the economy.

That is, $\lambda'(s) = \max\{\underbrace{\lambda_u'(s')}_{\beta/q\,\lambda}, \underbrace{\bar{\lambda}'(s')}_{\text{known given } k}\}$

° doesn't depend on λ.

- will have dependence on history until the point at which the constraint becomes binding



$\frac{\beta}{q} < 1$

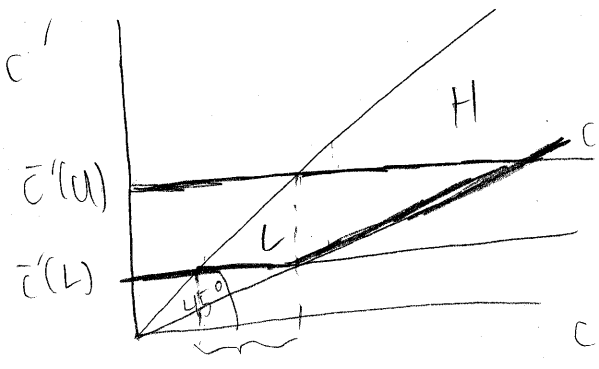
- consumption is just a simple transformation of λ.

This solves the model completely.

assume $u(c) = \frac{c^{1-\sigma}}{1-\sigma} \Rightarrow \lambda u'(c) = 1$

$\Rightarrow \lambda c^{-\sigma} = 1 \Rightarrow (\lambda)^{1/\sigma} = c$

$\Rightarrow c'_u(s') = (\lambda'_u(s'))^{1/\sigma} = \left(\frac{\beta}{q}\right)^{1/\sigma} \lambda^{1/\sigma} = \left(\frac{\beta}{q}\right)^{1/\sigma} c$



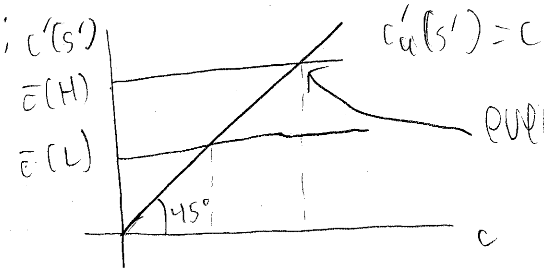
ergodic set.

◦ also:

$\bar{c}'(s') = [\bar{\lambda}(s')]^{1/\sigma}$

The best way to think of the solution to the model is to think about the evolution of λ.

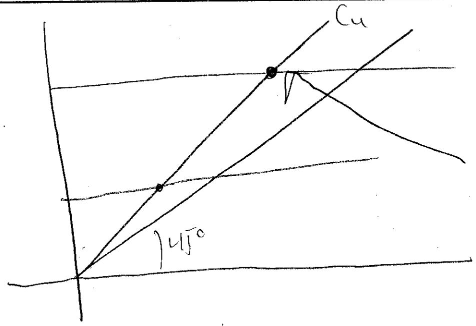
$\frac{\beta}{q} = 1$;



everyone will rise up to at least this level.

◦ consumption rises monotonically
◦ convergence to equality for everyone who starts with $c < \bar{c}(H)$.

$\frac{\beta}{q} > 1$



- you "outgrow these constraints."

GE: Technology (continuum of agents)

- linear storage:

  $C + a' = Ra + \bar{Y} \Rightarrow q = 1/R$

  aggregate consumption

- endowment economy (at steady state)

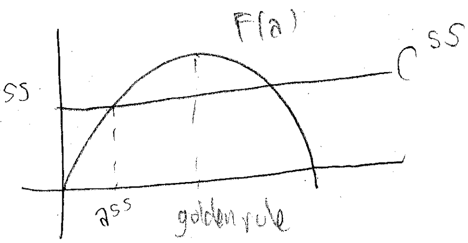
  $C \leq \bar{Y} \Rightarrow$ find q s.t. $C^{ss}(q^{ss}) = \bar{Y}$

- concave technology (Neoclassical)

  $C + a' \leq F(a)$, $F(\cdot)$ concave.

  $C^{ss} = F(a^{ss}) - a^{ss}$

  Need $\frac{1}{q} = F'(a^{ss})$



There are some opportunities for insurance.
Can think of this as having some sort of borrowing constraint: $u'(c_t) \leq R\beta E_t[u'(c_{t+1})]$

- can't really promise low consumption, because you would leave.