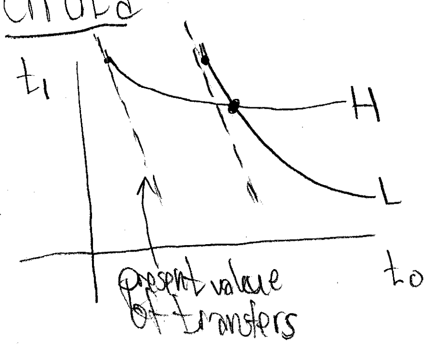
Errata



Private information economy with effort choices. Dynamic Planning problem

Look at the dual

$$K(v) = \min \sum_{t,s^t} q^t \underbrace{C}_{\text{inverse of u-fcn}}(u_t(s^t)) Pr[s^t | e^*]$$

$$\text{s.t.} \quad v = \sum_{t,s^t} \beta^t u(s^t) Pr[s^t | e^*] \quad (IR)$$

and (IC)

Endogenous state variable

- $V_{t-1}(s^{t-1}) \equiv \sum_{s=0}^{\infty} \beta^s E_{t-1}[u_{t+s}(s^{t+s}) | e^*]$
  - continuation utility/welfare (expected, discounted)
  - continuation lifetime utility on equilibrium path.

"One shot" deviations need to be ruled out:

$$V_{t-1}(s^{t-1}) \geq 1 + u(s^{t-1}, L) + \beta \underbrace{V_t(s^{t-1}, L)}_{\text{will resume equilibrium play}}$$

- This would be sufficient for (IC) to be satisfied if the game is continuous at infinity.

- Will get this if u is bdd or we have some sort of no-Ponzi condition.

Suppose we have a SP who is retiring after today and being replaced with another one who will have the job forever. He must tell the new one how much utility the agent is promised.

$$K(V) = \min_{u(s), V'} \sum_s \left(c(u(s)) + q K(V'(s)) \right) \pi(s | e^*)$$

$$\text{s.t. } V = \sum_s \left(u(s) + \beta V'(s) \right) \pi(s | e^*)$$

$$\text{and } V \geq 1 + u(L) + \beta V'(L)$$

$$\left. \begin{array}{l} V'(s) = g^v(v, s) \\ u(s) = g^u(v, s) \end{array} \right\} \text{optimal}$$

$\vdots$

$$u_t(s^t) = g^u(v_{t-1}(s^{t-1}), s_t)$$

$$V_t(s^t) = g^u(v_{t-1}(s^{t-1}), s_t)$$

$$v_{-1} = v \quad \text{given}$$

Let $u(c) = \log(c)$. Then $c(u) = \exp\{u\}$

"Guess and verify"

$K(v) = A \cdot \exp\{(1-\beta)v\}$

- can show this using the sequence problem.

<u>idea:</u> $v \Leftrightarrow \{u_t\}$ is optimal

$\hat{v} = v + \Delta \Leftrightarrow \{\underbrace{u_t + (1-\beta)\Delta}_{\hat{u}_t}\}$ is feasible.
(will satisfy IC and IR)

$$\Rightarrow \sum_t q^t E_{-1}[\exp\{\hat{u}_t\}]$$

$$= \sum_t q^t E_{-1}[\exp\{(1-\beta)\Delta\}\exp\{u_t\}]$$

$$= \exp\{(1-\beta)\Delta\} \underbrace{\sum_t q^t E_{-1}[\exp\{u_t\}]}_{=A}$$

Plug this into Bellman:

$$A\exp\{(1-\beta)v\} = K(v) = \min \sum_s (\exp\{u(s)\} + q A \exp\{(1-\beta)v'(s)\})\pi(s)$$

$$\text{s.t. } v = \sum_s (u(s) + \beta v'(s))\pi(s)$$

$$v \geq 1 + u(L) + \beta v'(s)$$

Let $\hat{u}(s) = u(s) - (1-\beta)v$

$\hat{v}'(s) = v'(s) - v$

$\Rightarrow$ constraints become:

$$0 = \sum_s (\hat{u}(s) + \beta\hat{v}'(s))\pi(s) \quad (1)$$

$$0 \geq 1 + \hat{u}(L) + \hat{v}'(L) \quad (2)$$

$\Rightarrow A = \min_{\tilde{u}, \tilde{v}'} \{ \exp\{\tilde{u}(s)\} + q\, A \exp\{\tilde{v}'(s)(1-\beta)\} \}$

s.t. (1) and (2). no state variable.

This can yield A as a fixed point and verifies that $K(v) = A\exp\{(1-\beta)v\}$

What does this say about g?

$g^u(v,s) = (1-\beta)v + \tilde{g}^u(s) \Rightarrow u(s) = (1-\beta)v'(s) + \underbrace{(1-\beta)g^v(s) + \tilde{g}^u(s)}_{\text{constant} = \bar{g}}$

$g^v(v,s) = v + \tilde{g}^v(s)$ This makes lifetime utility a random walk.

The FOCs of the problem are: (if we assume $\pi = 1/2$)

(u_H): $c'(u_H) = \lambda + \mu$

(v_H): $qK'(v_H) = \lambda\beta + \mu\beta$

$$\Rightarrow c'(u_H) = \frac{q}{\beta} K'(v_H)$$

(u_L): $c'(u_L) = \lambda - \mu$

(v_L): $qK'(v_L) = \beta\lambda - \beta\mu$

$$\Rightarrow c'(u_L) = \frac{q}{\beta} K'(v_L)$$

Envelope conditions:

$$K'(v) = \sum_s c'(u(s))\pi(s)$$

$$\Rightarrow K'(v(s)) = \sum_s c'(u(s'))\pi(s')$$

$$\Rightarrow c'(u(s)) = \frac{q}{\beta} \sum_{s'} c'(u(s'))\hat{\pi}(s')$$

This is really saying:

$$c'(g^u(v,s)) = \frac{q}{\beta} \sum_{s} c'(g^u(g^v(v,s),s'))\hat{\pi}(s')$$

$$\Leftrightarrow c'(u_t(s^t)) = \frac{q}{\beta} \sum_{s_{t+1}} c'(u_{t+1}(s^{t+1}))\hat{\pi}(s_{t+1})$$

Final version: $c' = \frac{1}{u'}$

$$\Leftrightarrow \frac{1}{u'_t(c_t)} = \frac{q}{\beta} E_t\left[\frac{1}{u'(c_{t+1})}\right]$$ (inverse Euler eqn)

$$= \frac{1}{R\beta} E_t\left[\frac{1}{u'(c_{t+1})}\right]$$ This is a different intertemporal equation.

We have learned:

1. There is history dependence (not autarky)
2. Inverse Euler equation holds $\Rightarrow$ Not income fluctuation model's allocation.

$$c_t = \exp\{(1-\beta)v_{t-1} + \tilde{g}^u(s)\} = \exp\{(1-\beta)v_t + \tilde{g}\}$$
$$= \exp\{\tilde{g}\}\exp\{\underbrace{(1-\beta)v_t}_{\text{random walk}}\}$$

$\Rightarrow c_t$ is a geometric random walk.

ie $c_t = a\, c_{t-1} \cdot \varepsilon_t$ $\quad \varepsilon_t$ iid with $E[\varepsilon_t] = 1$

There is consumption smoothing.

If $q=\beta$ $u=\log$ $\Rightarrow a=1$. By the inverse Euler eqn,
$c_t = E_t[c_{t+1}]$ ie $a=1$.

an implication of this model is that increasing inequality is efficient.

var($\log c_t$) growing linearly
$\Rightarrow c_t \to 0$ a.s. (immiseration)