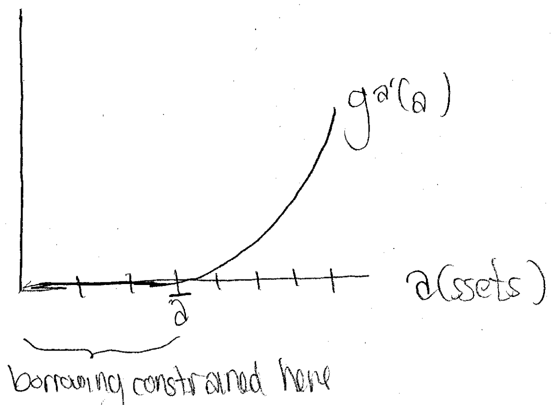

- can solve for critical value $\bar a$

Euler equation:

$$u'(Ra + y(s) + g^{a'}(a,s)) = R\beta E[u'(Rg^{a'}(a,s) + y(s') + g^{a'}(g^{a'}(a,s),s))|s]$$

where the individual does not save, we have: $g^{a'}(a,s)=0$

$$u'(R\bar a(s) + y(s)) = R\beta E[u'(y(s'))|s]$$

Suppose CRRA:

$$(R\bar a(s) + y(s))^{-\sigma} = R\beta[p_\ell(s) y_\ell^{-\sigma} + p_h(s) y_h^{-\sigma}]$$

$\Rightarrow \bar a(\ell), \bar a(h)$ - make sure both of these are on the grid.

GMM:

$$E[Z_t \underbrace{\varepsilon(\overbrace{g_t}^{\left(\frac{c_{t+1}}{c_t}\right)}, \overbrace{\theta}^{\binom{\beta}{\sigma}})}_{1-\beta R\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}}] = 0$$

Need $E[Z_t \underbrace{\frac{\partial \varepsilon}{\partial \theta'}}] \neq 0$.

in linear case, $\varepsilon = Y - X\beta$, so $\frac{\partial \varepsilon}{\partial \theta} = X$

Grids: σ 0 : 5 : 100
β 0.9 : 1 : 100

$$Q_n(\theta) = \bar{m}(\theta)' W^{-1} \bar{m}(\theta)$$

$$\hat{\theta} = \arg\min Q_n(\theta)$$

FOCs:

$$\underbrace{\frac{\partial m(\hat{\theta})}{\partial \theta}}_{\equiv H(\hat{\theta})\text{ - Jacobian}} \hat{W}^{-1} \bar{m}(\hat{\theta}) = 0$$

Taylor Expansion:

$$H(\hat{\theta})\hat{W}^{-1}[\bar{m}(\theta_0) + \overbrace{H(\theta^*)}^{\theta^* \in conv\{\hat{\theta}, \theta_0\}}(\hat{\theta} - \theta_0)] = 0$$

Rearranging,

$$\sqrt{n}(\hat{\theta} - \theta_0) = \underbrace{-[H(\hat{\theta})'\hat{W}^{-1}H(\theta^*)]^{-1}H(\hat{\theta})'\hat{W}^{-1}}_{\downarrow p \;\; -[H'W^{-1}H]^{-1}H'W^{-1}} \underbrace{\sqrt{n}\, m(\theta_0)}_{\downarrow d \;\; N(0, \Omega)}$$

$$\xrightarrow{d} N(0, \xi), \text{ where}$$

$$\xi = (H'W^{-1}H)^{-1}H'W^{-1}\Omega(W^{-1})'H(H'W^{-1}H)^{-1}$$

For the just id case:

$$\xi = H^{-1}W(H')^{-1}H'W^{-1}\Omega(W^{-1})'H H^{-1}W(H')^{-1}$$
$$= H^{-1}\Omega(H^{-1})'$$

Overid: Let $W = \Omega$

$\Xi = (H' W^{-1} H)^{-1}$

Income Fluctuations Problem

1] No uncertainty

$$\max_{\{c_t, a_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } c_t + a_{t+1} \leq R a_t + y_t \quad \forall t$$

$$\lim_{t \to \infty} a_t R^{-t} \geq 0$$

$$a_0 \text{ given}$$

Can rewrite budget constraint

$$\sum_{t=0}^{\infty} c_t R^{-t} \leq a_0 + \sum_{t=0}^{\infty} y_t R^{-t}$$

FOC: $u'(c_t) = \lambda (\beta R)^{-t}$

$\beta R = 1 \Rightarrow c_t = \bar{c} \quad \forall t$

where $c_t = \frac{r}{1+r}\left[a_0 + \sum_{t=0}^{\infty} y_t R^{-t}\right]$

2] Stochastic income with quadratic utility

$u(c) = \frac{1}{2}(c-b)^2$

$\Rightarrow u'''(c) = 0$

FOCs: (c_t): $\beta^t(c_t - b) = \lambda_t$

(c_{t+1}): $\beta^{t+1}(c_{t+1} - b) = \lambda_{t+1}$

(a_{t+1}): $\lambda_t = R E_t[\lambda_{t+1}]$

Euler equation: $c_t = R\beta E_t[c_{t+1}] + b(1-R\beta)$

If $\beta R = 1$, get a random walk: $c_t = E_t[c_{t+1}]$

3] Role of $u'''(\cdot)$

$$\max_s \underbrace{u(a_0 - s)}_{c_0} + \beta[p\,u(\underbrace{Rs - \overbrace{(1-b)D - pbD}^{\text{insurance}}}_{c_L}) + (1-p)\,u(\underbrace{Rs - pbD}_{c^{NL}})]$$

∘ what is the effect of b on savings in the first period?

FOC:

(c_0): $u'(c_0) = \beta R[p\,u'(c_L) + (1-p)\,u'(c^{NL})]$

$$\Rightarrow \frac{\partial s}{\partial b} = \frac{\beta R p(1-p) D[u''(c^{NL}) - u''(c_L)]}{\underbrace{SOC}_{<0}}$$

< 0 iff $u''(c^{NL}) > u''(c_L)$

Sufficient condition: $u'''(\cdot) > 0$ since $c^{NL} > c_L$

This is precautionary savings (prudence)

prudence $= -\dfrac{u'''(\cdot)}{u''(\cdot)} > 0 \rightarrow [\underbrace{-k}_{\text{lose for sure}}, \underbrace{\tilde{\varepsilon}}_{\text{mean zero rv}}] \succ [0, -k + \tilde{\varepsilon}]$

temperance $= -\dfrac{u''''(\cdot)}{u'''(\cdot)} > 0 \rightarrow [\hat{\varepsilon}_1, \hat{\varepsilon}_2] \succ [0, \hat{\varepsilon}_1 + \tilde{\varepsilon}_2]$

$E[\hat{\varepsilon}_1] = E[\hat{\varepsilon}_2] = 0$

⋮

$= -\dfrac{u^{(n)}(\cdot)}{u^{(n-1)}(\cdot)} > 0$

o prefer to have risk decomposed

Schlesinger / Eeckhoudt (AER '06).

4 CARA $u(c) = -e^{-\gamma c}$

$$V(x_0) = \max E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

$$\text{s.t. } c_t + a_{t+1} \leq \underbrace{Ra_t + y_t}_{\equiv x_t}$$

x_0 given

Let $\hat{c}_t = c_t - \frac{r}{R} x_0$

$\hat{a}_t = a_t - \frac{1}{R} x_0$

$\hat{x}_t = R\hat{a}_t + y_t = x_t - x_0$

$$\hat{c}_t + \frac{r}{R} x_0 + \hat{a}_{t+1} + \frac{1}{R} x_0 \leq \hat{x}_t + x_0$$

$$\Rightarrow \hat{c}_t + \hat{a}_{t+1} \leq \hat{x}_t$$

$$V(x_0) = \max E\left[\sum_{t=0}^{\infty} \beta^t u(\hat{c}_t + \tfrac{r}{R} x_0)\right]$$

$$\text{s.t. } \hat{c}_t + \hat{a}_{t+1} \leq \hat{x}_t$$

$$\hat{x}_0 = 0$$

with CARA, $u(\hat{c}_t + \frac{r}{R} x_0) = -u(\hat{c}_t)\, u(\frac{r}{R} x_0)$

$$V(x_0) = u(\tfrac{r}{R} x_0)\underbrace{\left[-\max E\left[\sum_{t=0}^{\infty} \beta^t u(\hat{c}_t)\right]\right] \text{ s.t. } \hat{c}_t + \hat{a}_{t+1} \leq \hat{x}_t,\ \hat{x}_0 = 0}_{\text{does not depend on } x_0}$$

$$= A u(\tfrac{r}{R} x_0)$$

The value function thus solves (assuming iid case

$$A u(\tfrac{r}{R} x_0) = \max_{a'} \left\{ u(x_0 - a') + \beta A E\left[u\left(\tfrac{r}{R}\underbrace{(Ra'(x_0) + \tilde{y})}_{=x'}\right)\right]\right\}$$

$$\Rightarrow A = \max_{a'} \left\{ u(\tfrac{x_0}{R} - a') + \beta A E\left[u(ra' \quad \tfrac{r}{R} x_0 + \tfrac{r}{R}\tilde{y})\right]\right.$$

FOC:

$$u'(\tfrac{x}{R} - a') = r\beta A E\left[u'(ra' - \tfrac{r}{R} x_0 + \tfrac{r}{R}\tilde{y})\right]$$

Recall that with cara: $u'(\cdot) = -\gamma u(\cdot)$

$$\Rightarrow u'\left(\frac{x_0}{R} - a'\right) = r\beta A\, E\left[u'\left(ra' - \frac{r}{R}x_0 + \frac{r}{R}\tilde{y}\right)\right] \quad (*)$$

Thus, $A = u\left(\frac{x}{R} - a'\right) + \frac{1}{r}u\left(\frac{x}{R} - a'\right)$

$$= \frac{R}{r}u\left(\frac{x_0}{R} - a'\right)$$

$$\Rightarrow V(x_0) = \frac{R}{r}u(c(x_0))$$

$$(*) \Rightarrow u'\left(\frac{x_0}{R} - a'\right) = r\beta \frac{R}{r} u'\left(\frac{x_0}{R} - a'\right) E\left[u'\left(ra' - \frac{r}{R}x_0 + \frac{r}{R}\tilde{y}\right)\right]$$

$$1 = \beta R\, u'\left(ra' - \frac{r}{R}x_0\right) E\left[u'\left(\frac{r}{R}\tilde{y}\right)\right]$$

$$\Rightarrow 0 = u'^{-1}(\beta R) + ra' - \frac{r}{R}x_0$$

$$- u'^{-1}\left(E\left[u'\left(\frac{r}{R}\tilde{y}\right)\right]\right)$$

$$\Rightarrow a'(x_0) = \frac{x_0}{R} \overbrace{- \frac{u'^{-1}(\beta R)}{r} + \frac{u'^{-1}\left(E\left[u'\left(\frac{r}{R}\tilde{y}\right)\right]\right)}{r}}^{-\bar{c}}$$

$$= \frac{x_0}{R} + \bar{c}$$

$$\Rightarrow c(x_0) = x_0 - a'(x_0) = \frac{r}{R}x_0 - \bar{c}$$

Thus, $a' = \frac{1}{R}(Ra + y) + \bar{c} = a + \frac{1}{R}\underset{\text{iid}}{y} + \bar{c}$ random walk with drift