What determines consumption and wealth, taking the income process as given?

Income Fluctuation Problem

Want to understand individual's problem before aggregating up.

$$c_t + a_{t+1} \le R a_t + y_t$$

$$c_t \ge 0, \quad a_t \ge -\varphi$$

Let $\hat{a} = a + \varphi$. Then,

$$c_t + \hat{a}_{t+1} \le R\hat{a}_t - r\varphi + y_t = z_t$$

$$c_t \ge 0, \quad \hat{a}_t \ge 0$$

$$v(z,s) = \max_{\hat{a}' \ge 0} \{ u(z - \hat{a}') + \beta E[v(R\hat{a}' + y(s') - r\varphi, s') | s] \}$$

iid case: $v(z,s)$ doesn't depend on s.

If $\beta R \ge 1$, then $c_t \to +\infty$

$a_t \to +\infty$

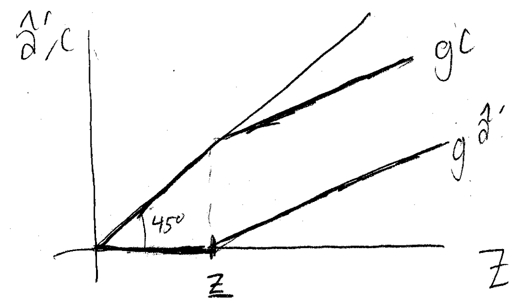
If $\beta R < 1$:

- V is increasing, concave, continuous.
- Looks like a two period problem

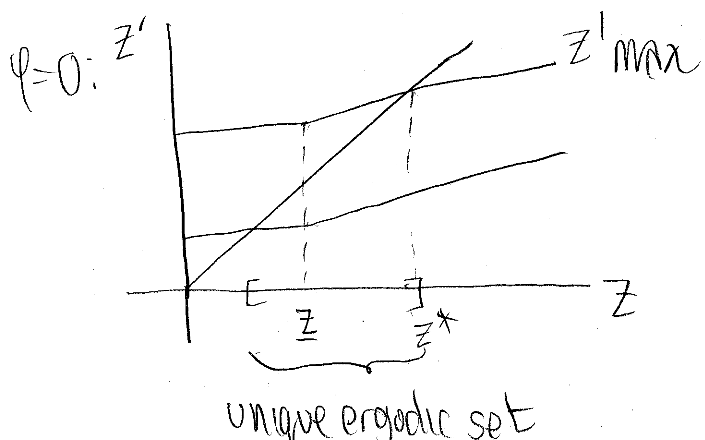
$\Rightarrow$ $\left.\begin{matrix} g^c \\ g^{\hat{a}'} \end{matrix}\right\}$ monotone nondecreasing in z.

∘ There exists $\underline{z}$ s.t. $g^{\hat{a}'}(z) = 0 \quad \forall z \le \underline{z}$

∘ $u'(\underline{z}) = \beta R \, E[u'(\underbrace{y - r\varphi}_{\text{no savings}})]$



$$z' = Rg^{\hat{a}'}(z) + y(s') - r\varphi$$



∘ In CARA case, there was no such upper bound.

Want to prove that $\exists \, z^*$ s.t.

$$z'_{max}(z) = Rg^{\hat{a}'}(z) + y_{max} \le z \quad \forall z \ge z^*$$

Proof: Let $c(z) = g^c(z)$

$$u'(c(z)) = \beta R \, E\left[\frac{u'(c(z'))}{u'(\bar{c}(z))}\right] u'(\bar{c}(z))$$

where $\bar{c}(z) = c(z'_{max}(z))$, $\underline{c}(z) = c(z'_{min}(z))$

If $\dfrac{E[u'(c(z'))]}{u'(\bar{c}(z))} \to 1$ as $z \to \infty$, we are done.

$$1 \le E\left[\frac{u'(c(z'))}{u'(c(z))}\right] \le \frac{u'(\underline{c}(z))}{u'(\bar{c}(z))}$$

since $g^{\hat{a}'} \uparrow$ $\longrightarrow \le y_{max} - y_{min}$

$$= \frac{u'(\bar{c}(z) - (\bar{c}(z) - \underline{c}(z)))}{u'(\bar{c}(z))} \le \frac{u'(\bar{c}(z) - (y_{max} - y_{min}))}{u'(\bar{c}(z))}$$

Claim: $\bar{c}(z) \to \infty$ as $z \to \infty$ (take this as given)

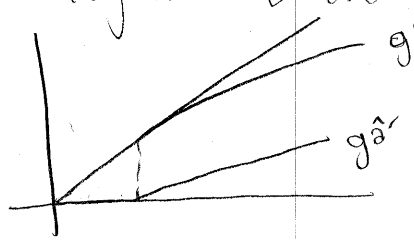
Lemma: given $-\dfrac{u''(x)}{u'(x)} \to 0$ as $x \to \infty$, $\dfrac{u'(x-A)}{u'(x)} \to 1$

since $\dfrac{u'(x-A)}{u'(x)} = \dfrac{u'(x)}{u'(x)} - \dfrac{u''(\tilde{x})A}{u'(x)}$, $\tilde{x} \in \text{conv}\{x-A, x\}$

$$\to 1 \text{ if } -\frac{u''(\tilde{x})}{u'(x)} \to 0 \quad \square$$

$u'(0) = \infty \Rightarrow$ will always save some. $(\hat{a}' > 0)$

MPC is higher than in PIH situation: $\left(\frac{r}{1+r}\right)$



$\leftarrow$ slope approaches $\frac{r}{1+r}$, since asymptotically risk neutral

Invariant Distributions

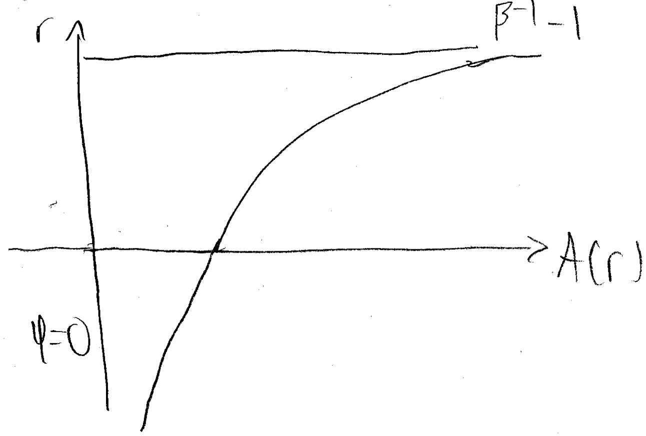
Initial distribution $F_0(z)$

Does $F_t(z) \to F(z)$ for some F?

- is F unique? is it stable? Yes, yes, and yes.

Let $A(r) = E[a'(z,r)]$ (what is expectation of invariant distribution?)

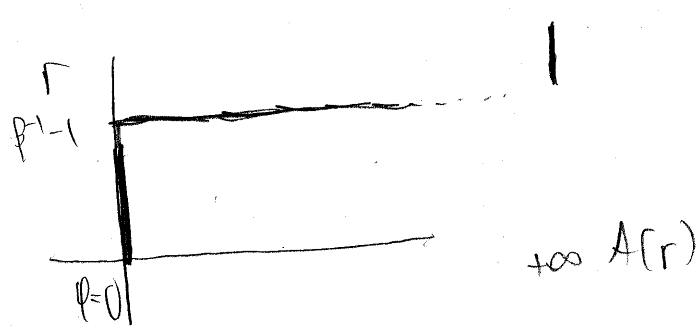
$A(\cdot)$ is not necessarily monotone in r, due to income and substitution effects.



$\varphi = 0$

typically, though, it is increasing.

with either complete markets or no uncertainty:



General Equilibrium effects

- adds capital
- $y_t = w l_t$, l_t is random, w economy-wide wage
- $N = \sum l^i p^i$ - total labor in the economy.

Aggregate technology:

$$C_t + K_{t+1} \leq F(K_t, N_t) + (1-\delta)K_t$$ (Just Neoclassical Growth model)

Steady state equilibrium:

(1) ∘ $\int g^{\hat{a}'}(z,r,w)\,dF(z;r,w) - \varphi = K$

∘ average level of assets in invariant distribution equals capital stock.

(2) ∘ $r = F_K(K,N) - \delta$

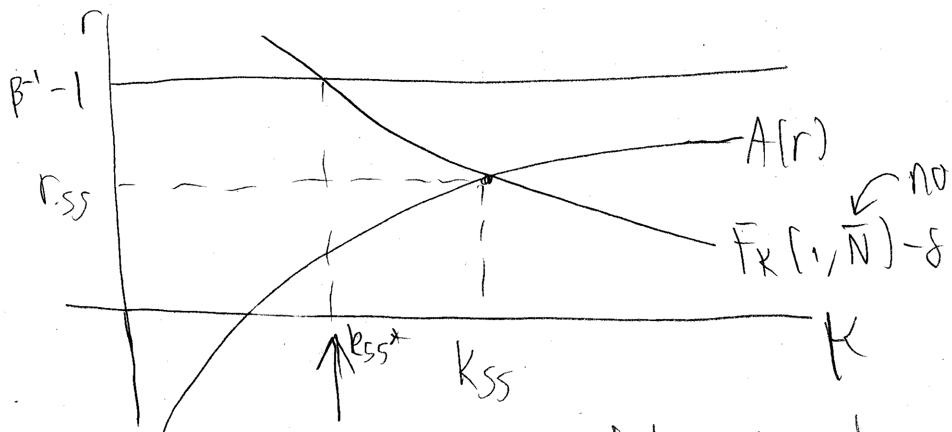
(3) ∘ $w = F_N(K,N)$

Three equations in three unknowns (K, r, w).

Use (2) to solve for $K(r)$. Plug into (3) to get $w(r)$. Plug both into (1) to get everything in terms of r.

$$A^{GE}(r) = \int g^{\hat{a}'}(z,r,w(r))\,dF(z;r,w(r)) - \varphi = K$$

- intersect this with $r = F_K(K,N) - \delta$



no shocks at the aggregate level.

with complete markets, steady state capital is here.

GE precautionary savings: $k_{ss}-k_{ss}^*$

- This is without assuming anything about the third derivative.
- Aiyagari is a must-read.
  - not much difference between this model and complete markets, unless you include some persistence in shocks.

Interest rate goes down if there is more risk, risk aversion, or persistence.

Can look at the cross-sectional distribution
- much more variation in income than in wealth.