LOP $\Rightarrow \exists$ stochastic discount factor

LOP + NA $\Rightarrow \exists$ a positive stochastic discount factor

LOP + NA + Complete markets $\Rightarrow \exists!$ positive stochastic discount factor

all theories about asset pricing boil down to

$$\underbrace{q}_{\text{price of fundamental assets}} = E[m \underbrace{p}_{\text{payoffs}}]$$

$$m = f(\underbrace{\text{data}}_{\text{realized at/by time } t}, \text{parameters})$$

Diagnostic tool gives us bounds. (Hansen-Jagannathan)

Special case: data on excess return (an asset with $q = 0$)

$$0 = \underbrace{E[mr]}_{=q} = E[m]E[\underbrace{r}_{\text{excess return}}] + \underbrace{\text{cov}(m,r)}_{\sigma_m \sigma_r \text{corr}(m,r)}$$

- We know that $\left|\frac{\text{cov}(m,r)}{\sigma_m \sigma_r}\right| \leq 1 \Leftrightarrow \left|\frac{E[m]E[r]}{\sigma_m \sigma_r}\right| \leq 1$

$$\Rightarrow \frac{\overbrace{\sigma_r}^{\text{in the data}}}{|E[r]|} \leq \frac{\overbrace{\sigma_m}^{\text{need this to be volatile}}}{|E[m]|}$$

- CCAPM fails this bound, since $\sigma_m \approx 0$ as consumption is very smooth

More generally,

regress $\underbrace{y}_{\text{stochastic discount factor}} = a + \overbrace{x'}^{\text{payoffs}} b + e$

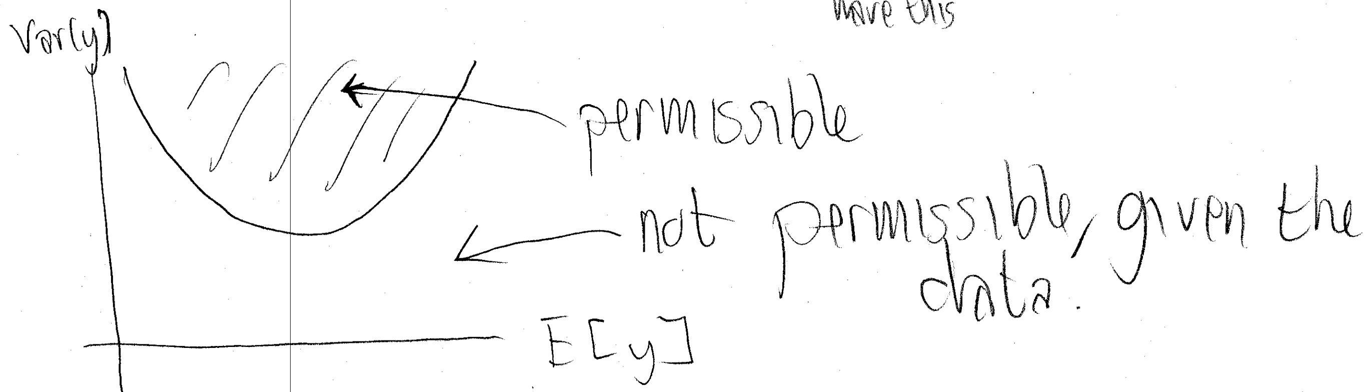
$$\mathrm{Cov}(e,x)=0 \Rightarrow \underbrace{\mathrm{Var}(y)}_{\text{var of stochastic discount factor}} \geq \mathrm{var}(x'b)$$

Convenient, since $b = [\mathrm{cov}(x,x)]^{-1}\mathrm{cov}(x,y)$

$a = E[y] - E[x'b]$

and $\mathrm{cov}(x,y) = q - E[y]E[x]$

$$\Rightarrow b = \underbrace{[\mathrm{cov}(x,x)]^{-1}}_{\text{from data}}[q - \underbrace{E[y]}_{\text{don't have this}}E[x]]$$

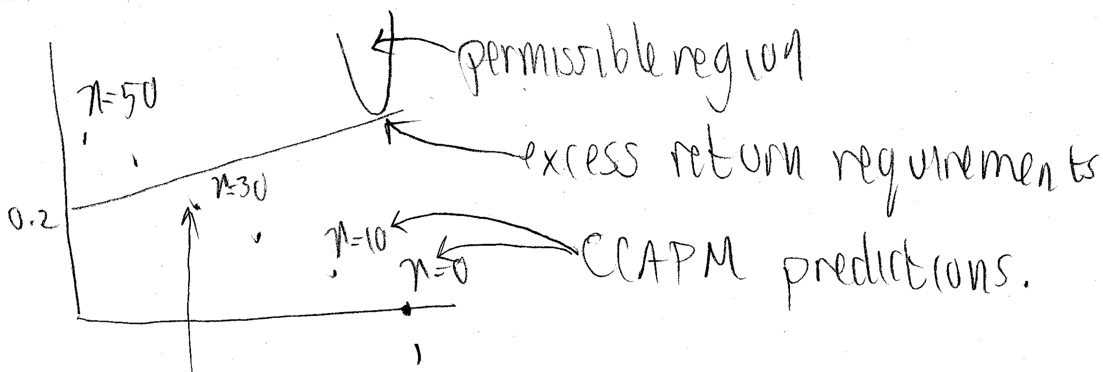


- Have b as a function of $E[y]$ and a as a function of $E[y]$
- can create bounds.

If we have a risk-free asset: $q = E[m \cdot r]$

$= r\,E[m]$

$\Rightarrow E[m] = \frac{q}{r}$



around $\eta=30$, the excess return puzzle is almost satisfied.

The stochastic discount factor just isn't volatile enough for the CCAPM model to satisfy the bounds. (unless we amplify everything by increasing risk aversion.)

Other approaches:

Exotic preferences:

- Risk-aversion vs. IES (Epstein-Zin)
- "First-order risk aversion" (Epstein-Zin) - with expected utility, risk aversion is a second-order phenomenon. (can do it with EU, but need kinks everywhere.)
- Habits: $u(c_t, c_{t-1})$ (Abel/Campbell-Cochrane)
  e.g. $u(c_t - \alpha c_{t+1})$, $\alpha \approx 1$, $\alpha < 1$
  - these mess up other things.

Heterogeneous agents and incomplete markets

- Uninsured idiosyncratic risk
  - conditional variance of consumption growth at th

idiosyncratic level matters.
- Private information and limited commitment

Knightian uncertainty
- afraid of not understanding returns.
- robust control from the engineering literature.

Others say that risk premium has gone away
- past excess returns were just unexpected shocks (ie tax reform)
- bonds are more liquid
- thick tails (Weitzman) - big, though infrequent, crashes.
  - using just the post-war data does worse
  - should include the great depression

Can you infer the costs of business cycle by looking at stock returns without marrying yourself to a model?
- estimate stochastic discount factor and then using it to price an asset that counters business cycles.

Incomplete markets
- uncertainty in income, but can invest in a single risk-free asset. They do not have enough assets to "span the uncertainty."

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t. $A_{t+1} = (1+r)(A_t + y_t - c_t)$

$$\Rightarrow c_t = \frac{r}{1+r}\left[A_t + y_t + \sum_{j=1}^{\infty} R^{-t} y_{t+j}\right]$$

- "constant consumption" (depends on present value of income) - more like consumption smoothing
- consume the annuity value of the present value.

Uncertainty: "add some expectations" (tempting)

$$(1) \quad c_t = \frac{r}{1+r}\left[A_t + y_t + E_t\left[\sum_{j=1}^{\infty} \left(\frac{1}{1+r}\right)^t y_{t+j}\right]\right]$$

- this assumes certainty equivalence
- valid if $u(c)$ is quadratic and $c \in \mathbb{R}$ (linear Euler equations)

Main insight: permanent income: $y_t^p = y_t + E_t\left[\sum_{j=1}^{\infty} \left(\frac{1}{1+r}\right)^t y_{t+j}\right]$

- c_t is a function of y_t^p, not the individual y_t

$$(1) \Rightarrow \Delta c_t = c_t - c_{t-1} = \frac{r}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \left[E_t[y_{t+j}] - E_{t-1}[y_{t+j}]\right]$$

$$\Rightarrow E_{t-1}[\Delta c_t] = 0$$

Consumption is a random walk conditional on $\mathcal{F}_{t-1}$

No insurance here, but consumption smoothing will minimize Δc_t.