Townsend: There is evidence of risk-sharing, but we can reject the hypothesis of perfect risk-sharing.

(*) Why don't countries share risk better?

(*) How good is the representative agent model an approximation when risk sharing is almost perfect?

$c^i = g^i(C) \Rightarrow C_J \equiv \sum_{i\in J} c^i = \sum_{i \in J} g^i(C) \equiv G^J(C)$, where $J \subseteq I$ is a sub group of the population.

Complete market model

Every good at a different time/state is a different good. Complete markets $\Rightarrow$ one budget constraint.

Consumers: $\max \sum_{t,s^t} \beta^t u^i(c_t^i(s_t)) \Pr[s^t]$

trade occurs at time 0.

$$\text{s.t.} \quad \sum_{t,s^t} q_t^0(s^t)\left(c_t^i(s^t) - y_t^i(s^t)\right) \leq 0 \quad (\mu^i)$$

FOC: $(c_t^i(s^t))$: $\beta^t u^{i\prime}(c_t^i(s^t)) \Pr[s^t] = \mu^i q_t^i(s^t)$

Note: $q_t^0(s^t)$ is the price of a good delivered at time t in state s^t when trade occurs at $t=0$

Definitions:

- price system: $q^0 = \{q_t^0(s^t)\}$
- allocation: $C = \{c_t^i(s^t)\}$

- feasibility: $\sum_{i \in I} c_t^i(s^t) = \sum_{i \in I} y_t^i(s^t)$

a <u>competitive equilibrium</u> is (q°, c) such that

- i chooses c^i optimally given q°
- markets clear: $\sum_{i \in I} c_t^i(s^t) = \sum_{i \in I} y_t^i(s^t)$ (ie the allocation is feasible)

The FOCs give us:

$$\frac{u^{i\prime}(c_t^i(s^t))}{u^{1\prime}(c_t^1(s^t))} = \frac{\mu^i}{\mu^1} \quad \forall i \qquad \left(\text{cf } \frac{\lambda^1}{\lambda^i}\right)$$

- MU ratios are kept constant across time and states
- This is the same as the Pareto problem

The CE (competitive equilibrium) allocation is Pareto optimal. (First Welfare Theorem)

The second welfare theorem tells us that, under some convexity assumptions, there exist some transfers and prices for any PO allocation such that the PO allocation becomes the CE allocation.

Asset pricing: (just a parenthetical for now)

Let $\underbrace{\{d_t^j(s^t)\}}$ be the dividend stream for asset j

how much i will get paid in each state at each time

If we normalize $q_0^0(s^0)=1$, then the value of the dividend stream is:

$$p^0(s_0)=\sum_{t,s^t} q_t^0(s^t)\, d_t^j(s^t)$$

Note that $$\frac{q_t^0(s^t)}{q_0^0(s^t)} = \frac{\beta^t u^{i\prime}(c_t^i(s^t))\Pr[s^t]}{u^{i\prime}(c_0^i(s^0))\underbrace{\Pr[s^0]}_{=1}}$$ (relative price equals MRS^i)

$$= \beta^t \frac{U'(c_t(s^t))}{U'(c_0(s^0))}\Pr[s^t]$$

in endowment economy, agg. cons. is equal to agg. wealth

$$= \beta^t \frac{U'(Y_t(s^t))}{U'(Y_0(s^0))}\Pr[s^t]$$

This is the Breeden-Lucas "Tree Model"

Thus, $$p_j^0(s_0)=\sum_{t,s^t}\beta^t \frac{U'(Y_t(s^t))}{U'(Y_0(s^t))}\Pr[s^t]\, d_t^j(s^t)$$

Arrow Sequential Trading Implementation

sequential budget constraint

$$c_t(s^t)-y_t(s^t)+\sum_{s_{t+1}} q(\underbrace{s^t}_{\text{what market you are in}},\underbrace{s_{t+1}}_{\text{what assets you are buying}})\, a_{t+1}(s^t,s_{t+1}) \le \underbrace{a_t(s^t)}_{\text{quantity of claims to asset }(t,s^t)}$$

Need $c_t(s^t) \geq 0$ and No Ponzi condition: $\lim_{t\to+\infty} \sum_{s^t} q_t^0(s^t) a_t(s^t) \geq 0$

This is then equivalent to

$$\sum_{t,s^t} q_t^0(s^t)\left(c_t(s^t) - y_t(s^t)\right) \leq a_0(s_0)$$

To see why, let $\dfrac{q_\tau^0(s^\tau)}{q_t^0(s^t)} = q_\tau^t(s^\tau)$

Then $q(s^t, s_{t+1}) = q_{t+1}^t(s^{t+1}) = q_{t+1}^t(s^t, s_{t+1})$

also, $q_t^0(s^t) = q(s_0, s_1)\, q(s^1, s_2) \cdots q(s^{t-1}, s_t)$

$$a_0(s_0) \geq c_0(s_0) - y_0(s_0) + \sum_{s_1} q(s_0, s_1)\, a_1(s_0, s_1)$$

$$\boxed{a_1(s^1)} \geq c_1(s^1) - y_1(s^1) + \sum_{s_2} q(s^1, s_2)\, a_2(s^1, s_2)$$

$\vdots$

$$a_t(s^t) \geq c_t(s^t) - y_t(s^t) + \sum_{s_{t+1}} q(s^t, s_{t+1})\, a_{t+1}(s^t, s_{t+1})$$

$$\Rightarrow a_0(s_0) \geq c_0(s_0) - y_0(s_0) + \sum_{s_1} q(s_0, s_1)\left[c_1(s^1) - y_1(s^1)\right] + \sum_{s^2} q_2^0(s^2)\, a_2(s^2) + \ldots$$

No Ponzi gives us:

$$\sum_{t,s^t} q_t^0(s^t)\left(c_t(s^t) - y_t(s^t)\right) \leq a_0(s^0)$$

- Let $A_t(s^t) = \sum_{\tau=0}^{\infty} q_{t+\tau}^{t}(s^t, s^\tau) y_t(s^t, s^\tau)$ value of future income stream

<u>Natural borrowing limit</u>: $a_{t+1}(s^t, s_{t+1}) \geq -A_{t+1}(s^{t+1})$

$\left(\Leftrightarrow \underbrace{a_{t+1}(s^t, s_{t+1})}_{\text{what you borrow}} + \underbrace{A_{t+1}(s^{t+1})}_{\text{what you are worth}} \geq 0 \right)$

∘ you don't go into debt that is impossible to pay off

Complete markets $\Leftrightarrow$ P.O. and perfect risk sharing