Lecture notes will be provided after class.

Last time: found that there were small costs to consumption volatility (business cycles)
- The magnitudes of idiosyncratic uncertainty are bigger and can thus have much larger effects
- This class will focus on idiosyncratic uncertainty.

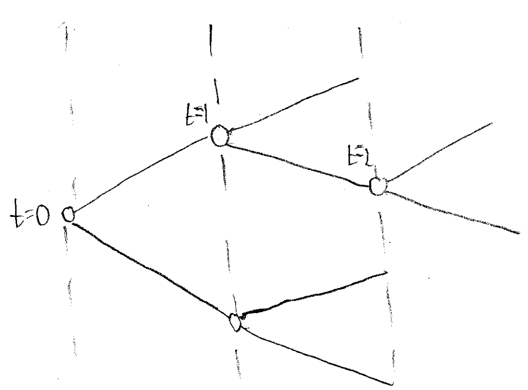
Pareto optimal allocation subject to idiosyncratic uncertainty?
- Market counterpart: complete markets
- Useful as a benchmark

This course is primarily about consumption

Let there be I agents w/preferences

$$\sum_{t,s^t} \beta^t u^i(c_t^i(s^t))\Pr(s^t)$$

- uncertainty: at time t, state of world, s_t, is realized. Assume $s_t \in S$ with $\underbrace{|S| < +\infty}_{\text{sidesteps measure theory}}$



- we start at $t=0$, since this is a macro course
- $s^t = (s_0, s_1, \ldots, s_t)$ is the history up time t

$s_t \in S^t = S \times \ldots \times S$ ($t+1$ times)

Final description of uncertainty: $Pr[s^t]$

Let $\underbrace{x_t(s^t)}$ be a function. Then, $E_0[x_t] = \sum_{s^t \in S^t} x_t(s^t) Pr[s^t]$

$x_t : S^t \to \mathbb{R}$

An allocation is a sequence of consumption functions $\{c_t(s^t)\}_{t=0}^{\infty}$

- $u' > 0$, $u'' < 0$, differentiable and satisfies inada condition

at time t, s_t determines each person's income: $y^i(s_t)$. Define $Y(s_t) = \sum_{i \in I} y^i(s_t)$.

- e.g. no aggregate uncertainty: $Y(s^t) = Y \ \forall s^t \in S^t$
- e.g. $y^i(s_t) = y(s_t) \ \forall i \Rightarrow Y(s_t) = |I| \, y(s_t)$

The social planner's problem (Pareto problem v.1.0)

$$\max_{\{c_t^i(s^t)\}} \sum_{i \in I} \underbrace{\lambda^i}_{\text{Negishi weights}} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t u^i(c_t^i(s^t)) Pr[s^t]$$

$$\text{s.t.} \quad \sum_{i \in I} y^i(s_t) \geq \sum_{i \in I} c_t^i(s^t) \qquad \underbrace{\forall t, s^t}_{\text{this is a lot of contraints}}$$

- can trace out all PO points by varying the weights $\{\lambda^i\}$, since this a convex problem

Pareto prob. v.1.1: $\displaystyle \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \max_{\{c_t^i(s^t)\}} \left[\sum_{i \in I} \lambda^i u^i(c_t^i(s^t)) \right] Pr[s^t]$

$$\text{s.t.} \quad Y(s_t) \geq \sum_{i \in I} c_t^i(s^t) \qquad \forall t, s^t$$

- The solution to this problem in each period/state depends only on output and thus only on the state: $c_t^i(s^t) = c_t^i(s_t) = g^i(Y(s_t))$ for some function $g = g^1 \times \dots \times g^I$

Perfect risk sharing here is imposed by the Pareto problem. (also implies history independence.)

Can solve V1.0 by Lagrangians.

$$\mathcal{L} = \sum_{i \in I} \lambda^i \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t u^i(c_t^i(s^t)) \Pr[s^t] + \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \theta_t(s^t)$$

FOCs:

$$(c^i(s^t)): \quad \beta^t \lambda^i u^{i\prime}(c^i(s^t)) \Pr[s^t] = \theta_t(s^t)$$

$$\Rightarrow \quad \frac{u^{i\prime}(c^i(s^t))}{u^{1\prime}(c^1(s^t))} = \frac{\lambda^1}{\lambda^i} \Rightarrow c^i(s^t) = \frac{\lambda^1}{\lambda^i}(u^{i\prime})^{-1}(u^{1\prime}(c^1(s^t)))$$

- ratio of MU's must be in some proportion
- if i's consumption increases, j's must increase as well
- $c^i(s^t)$ increases in $c^1(s^t)$
- Feasibility pins down $c^1(s^t)$

CARA: $u^i(c) = -\frac{1}{\gamma^i}\exp\{-\gamma^i c^i\}$, then

$$\lambda^i \exp\{-\gamma^i c^i\} = \lambda^1 \exp\{-\gamma^1 c^1\}$$

$$\Rightarrow \exp\{\gamma^1 c^1 - \gamma^i c^i\} = \frac{\lambda^1}{\lambda^i}$$

$$\Rightarrow \gamma^1 c^1 - \gamma^i c^i = \log\left(\frac{\lambda^1}{\lambda^i}\right) \equiv B^{1,i}$$

$$\Rightarrow c^i = -\frac{B^{1,i}}{\gamma^i} + \frac{\gamma^1}{\gamma^i} c^1$$

$$c^i = -\frac{B^{1,i}}{\gamma} + c^1 \quad \text{if } \gamma^i = \gamma \ \forall i$$

Here c^i increases in λ^i, as expected.

$$Y = \sum_{i\in I} c^i = -\underbrace{\sum_{i\in I} \frac{1}{\gamma^i} \log\left(\frac{\lambda^1}{\lambda^i}\right)}_{B} + \gamma^1 c^1 \sum_{i\in I} \frac{1}{\gamma^i}$$

$$\Rightarrow \gamma^1 c^1 = \frac{1}{\sum_{i\in I} \frac{1}{\gamma^i}} (Y + B)$$

$$\Rightarrow c^i = g^i(Y) = \frac{\frac{1}{\gamma^i}}{\sum_{i\in I} \frac{1}{\gamma^i}} Y + \tilde{B}^i$$

Since $\sum_{i\in I} c^i = Y$, $\sum_{i\in I} \tilde{B}^i = 0$

Pareto problem V 2.0 with capital and L=1

- Production function $F(k_t, \underbrace{n_t}_{=1\,\forall t}, s_t)$

$$\max \sum_{i\in I} \lambda_i \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t u^i(c_t^i(s^t)) \Pr[s^t]$$

s.t.,

$$F(k_t(s^{t-1}), 1, s^t) + (1-\delta)k_t(s^{t-1}) - k_{t+1}(s^t) = \sum_{i\in I} c_t^i(s^t)$$

for all t, s^t.

- savings decision here. Last time was special case with $F(s^t)$, $\delta = 1$, and $k_{t+1}^* = 0\ \forall t$.

Version 2.1: Two stage problem

$$\max_{c, k'} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \Pr[s^t] \max_{c^i} \sum_{i\in I} \lambda^i u^i(c_t^i(s^t)))$$

$$\text{s.t. } C_t(s^t) = \sum_{i\in I} c_t^i(s^t)$$

$$\text{s.t. } F(k_t(s^{t-1}), 1, s_t) + (1-\delta)k_t(s^{t-1}) - k_{t+1}(s^t) = C_t(s^t)$$

- Perfect risk sharing $\Rightarrow$ consumption is a function of aggregate consumption, not aggregate output as in previous problem: $c^i(s^t) = g^i(C_t(s^t))$ vs. $c^i(s^t) = \bar{Y}(s^t)$

- empirical test: $c^i(s^t) = \alpha + \beta g^i(C_t(s^t)) + \sum \gamma + \varepsilon$
  - theory suggests: $\gamma = \alpha = 0$

Define $U(C_t(s^t)) \equiv \max_{c^i} \sum_{i \in I} \lambda^i u^i(c^i)$

$$\text{s.t.} \quad \sum_{i \in I} c^i = C_t(s^t)$$

$$\Rightarrow \max_{C, k'} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} U(C_t(s^t))$$

$$\text{s.t.} \quad F(k_t(s^{t-1}), 1, s_t) + (1-\delta)k_t(s^{t-1}) - k_{t+1}(s^t) = C_t(s^t)$$

Heterogeneous agents are isomorphic to a representative agent, efficiency, and complete markets.

Testing perfect insurance

(*) READ Townsend's paper! (the EMA one)

- Finds significant idiosyncratic risk
- Tests: $c_t^i = \underbrace{\alpha^i}_{\text{should} = 1} C + \underbrace{\beta y_t^i}_{\text{should} = 0}$

EMA paper: finds α^i close to 1 and $\beta \neq 0$

- could reject perfect insurance, but there was definitely evidence supporting insurance