Problem sets due Friday.
Final: October 25th
Welfare costs of business cycles turns out to be small.

Reading is expected. There are a lot of open questions

Intertemporal preferences

a) $t = 1, 2, \ldots, T$ dates
$U[c_1, c_2, \ldots, c_T]$ preferences over consumption stream

b) $t = 1, 2, \ldots$
$U[c_1, c_2, \ldots]$

Examples:

- $U[c_1, \ldots, c_T] = \sum_{t=1}^{T} u_t(c_t)$
  - additive separability. Removes some interesting linkages among consumption in different periods
  - rules out habits and some aspects of durable goods
  - if time periods are short, this can be a bad approximation
- $U[c_1, \ldots, c_T] = \sum_{t=1}^{T} u_t(c_t, s_t)$
  - e.g. $s_t = f(c_t, s_{t-1})$, s_0 given
    - $u_t(c_t, s_t) = u[c_t - \alpha s_t]$, $s_t = \rho c_t + (1-\rho) s_{t-1}$
    - habit formation

- This introduces a good amount of flexibility.
- $U[c_1,\ldots,c_T] = \sum_{t=1}^{T} \beta^{t-1} u_t(c_t)$

Uncertainty

Expected utility - (if we accept some restrictions)

Suppose $(\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_T)$ is a stochastic process

$$E[u[\tilde{c}_1,\ldots,\tilde{c}_T]] \equiv V[\tilde{c}_1,\ldots,\tilde{c}_T]$$

Example:

- $\underbrace{E\left[\sum_{t=1}^{T} \beta^{t-1} u(\tilde{c}_t)\right]}$ — this ties together risk aversion and intertemporal substitution

$\left(E\left[G\left(\sum_{t=1}^{T} \beta^{t-1} u(\tilde{c}_t)\right)\right]?\right)$

- Here, risk aversion will depend on curvature of u and of G.

Kreps-Porteus and Epstein-Zin try to relax this link.

Level versus Growth versus Uncertainty

assume:
- $E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$
- $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$ CRRA

Exercise (Lucas '87)

Suppose:

$c_t = (1+\lambda)(1+g)^t \quad t = 0, 1, \ldots$ • certainty

Want to compare effects of λ and g.
- is 451 or 452 more important? (when we introduce uncertainty)

$$V(\lambda, g) = \sum_{t=0}^{\infty} \beta^t \frac{((1+\lambda)(1+g)^t)^{1-\sigma}}{1-\sigma}$$

$$= \frac{(1+\lambda)^{1-\sigma}}{1-\sigma} \sum_{t=0}^{\infty} (\beta(1+g)^{1-\sigma})^t$$

$$= \frac{(1+\lambda)^{1-\sigma}}{1-\sigma} \frac{1}{1-\beta(1+g)^{1-\sigma}}$$

Want to compare $(\underbrace{\lambda_0}_{\equiv 0}, g_0)$ and (λ_1, g_1)
- When do we have indifference?
- How do we value levels vs. growth?

$$V(1, g_0) = V(\lambda_1, g_1)$$

$$\Rightarrow \frac{1^{1-\sigma}}{1-\sigma} \frac{1}{1-\beta(1+g_0)^{1-\sigma}} = \frac{(1+\lambda_1)^{1-\sigma}}{1-\sigma} \frac{1}{1-\beta(1+g_1)^{1-\sigma}}$$

$$\Rightarrow \lambda_1 = \left[\frac{1-\beta(1+g_1)^{1-\sigma}}{1-\beta(1+g_0)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} - 1$$

$$\approx \frac{g_0 - g_1}{\rho - (1-\sigma)g_0} \qquad \underbrace{\beta}_{\text{discount factor}} = e^{-\overbrace{\rho}^{\text{discount rate}}}$$

$$\lambda_1 \approx \frac{g_0 - g_1}{r - g_0}, \qquad \underbrace{r = \rho + \sigma g_0}_{\text{interest rate}}$$ is this in growth models.

- This is what we would get from a present value calculation.

$C_t = (1+\lambda)(1+g)^t c_0 \varepsilon_t \qquad t = 0, 1, \ldots$

- ε_t stochastic, $E[\varepsilon_t] = 1$
- e.g. $C_t = (1+\lambda)(1+g)^t c_0 \exp\{-\frac{\sigma^2}{2}\} z_t$

  $\log c_t = \log(1+\lambda) + t\log(1+g) + \log c_0 + \log \varepsilon_t$,
  where $\log \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

Remark: $E[\log(z)] < \log E[z]$ — Jensen's inequality

- Estimates give $\hat{\sigma}_\varepsilon \approx 0.027$

Let $g = 0$:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}\right] = \frac{1}{1-\beta} \frac{1}{1-\sigma}\left[(1+\lambda)\exp\left\{-\sigma \frac{\sigma_\varepsilon^2}{2}\right\}\right]^{1-\sigma}$$

Comparing this to $\lambda = 0, \sigma_\varepsilon^2 = 0$ case, the indifference curve looks like:

$$\left[(1+\lambda)\exp\left\{-\sigma \frac{\sigma_\varepsilon^2}{2}\right\}\right]^{1-\sigma} = 1$$

$$\Rightarrow \log(1+\lambda) = \sigma \frac{\sigma_\varepsilon^2}{2}$$

$$\Rightarrow \lambda \approx \sigma \frac{\sigma_\varepsilon^2}{2}$$

Growth seems to matter more than variance in consumption, according to this exercise