## Problem sets

Here are the ground rules for problem sets:

**The solutions** are to be written out carefully and legibly, in good mathematical style.

(1) "Careful" has an obvious meaning.

(2) "Legibly" means do it in LaTeX or if handwritten, in ink or dark pencil, double-spaced. Any paper that does not meet this criterion will be handed back for you to do over.

(3) "Good mathematical style" means the style required by editors of math journals. Attached is a summary of what this means.

**Note** Your first written solutions constitute a first draft. Plan to rewrite them to clean up grammar and punctuation and sentence structure before handing ~~the~~ paper in.

The first problem set has only 2 problems that have to be written up. (The other two are "answers only.") So this should not be too arduous.

**Collaboration** You are encouraged to collaborate on the daily homework. (i.e., the "exercises.")

But the work on the problem sets is to be strictly your own. If you can't do all of a problem, do what you can and write "here's where I got stuck." Don't fake it if you can't do it!

**Late papers** are not accepted - hand in what you've done

# Comments on style

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To write in good mathematical style, you should:

(1) Write in complete sentences.

(2) Punctuate! (Correctly, if possible.)

(3) Write legibly. (An illegible proof is incorrect by definition.)

(4) Avoid the "stream of consciousness" style popularized by Wm. Faulkner. When you finish a thought, stop, put down a period, and take a good breath before you begin the next sentence (with a capital letter, please).

(5) Steer a middle ground between too much detail and not enough. Give reasons for your answer sufficient to convince your reader (in this case, the grader) that your argument is correct and you know why it is correct. But don't fill the pages by checking each tiny detail in writing; it only bores your reader and gives you writer's cramp.

At one extreme of style are those sparsely-written texts (such as Rudin) that require the reader to ponder each sentence and fill in most of the details. At the other extreme are those solutions written by your most conscientious fellow-student, so full of details that the basic idea is invisible!

Try to hit somewhere in the middle.

(6) Don't use mathematical symbols as parts of speech in an ordinary sentence. Bad examples: "Consider the set of all numbers $< 1$." "Consider the $\cap$ of the sets A and B." "Consider a function f mapping $A \to B$." Here is how to write these sentences correctly:

Consider the set of all numbers less than 1.

or

Consider the set of all numbers x such that $x < 1$.

Consider the intersection of the sets A and B.

or

Consider the set $A \cap B$.

Consider a function f mapping A into B.

or

Consider a function $f : A \to B$.

(7) Don't use logical symbols <u>at all</u>. The symbols $\exists$, $\ni$, $\forall$, $\exists!$, $\vee$, $\wedge$ as well as the abbreviations s.t., w.r. to, are to be avoided in mathematical writing. In papers in <u>logic</u>, these symbols constitute part of the subject matter and are completely appropriate. In informal mathematical discourse, on blackboard or paper, they are often used as "parts of speech", in a sort of mathematical shorthand. However, they are not allowed by editors in formal mathematical writing.

Just as you wouldn't submit a history paper that is written partly in secretarial shorthand, don't submit a math paper written partly in mathematical shorthand!

(8) One exception to the rule in (7) is the use of the symbols $\Rightarrow$ (implies) and $\Leftarrow$ (is implied by) and $\Leftrightarrow$ (is equivalent to). One of course does not use these symbols as word-substitutes, any more than one uses $<$ or $+$ or $\cap$ as word-substitutes. But usage is allowed in phrases such as: "We show that (a) $\Rightarrow$ (b) $\Rightarrow$ (c)," or "To show (a) and (b) are equivalent, it suffices to show that (a) $\Rightarrow$ (b) and (b) $\Rightarrow$ (a)".

There is a reason why editors (at least those who are also mathematicians) enforce rule (7) strictly. Most mathematical readers find sentences in which this rule is violated quite unreadable, just as they find secretarial shorthand unreadable. They translate the sentence into the English language (or German, or French, or ...) mentally, before attempting to understand it.

Occasionally a text-book editor (who is usually not a mathematician) will let an author get away with violating these rules. Here is a horrendous example, quoted from a well-known text, <u>verbatim</u>:

Let $f : [0,\Omega) \to [0,\Omega)$ be s.t. $f(\alpha) < \alpha$ for all $\alpha \geq$ some $\alpha_0$. Then $\exists\, \beta_0\ \forall\, \beta\ \exists\, \alpha \geq \beta : f(x) \leq \beta_0$.

Some people can grasp the meaning of these two sentences immediately; most mathematicians can't!