14.731: Economic History

Without Consent but With Contract: Incentives, Monitoring, and Manumission in Louisiana, 1719-1820

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Abstract

This paper develops a simple contract theoretic model to help explain the economic incentives involved in the manumission of slaves. The key ideas of the model are three fold. First, the individual rationality constraint is replaced by a survival constraint, a constraint which, in effect, prevents the master from rationally killing his slave, providing an asymmetry between reward and punishment. Secondly, I model monitoring costs. Finally, consistent with the fact that slaves in Louisiana were primarily housed in barracks, I allow for endogenous choice of the "living standards" that a master will provide for his slaves. The economies of scale present in housing ensures that masters of larger plantations have more to take away from slaves in punishment and will therefore not need to resort to positive rewards as much as masters of smaller plantations.

Two empirical predictions of this model are confirmed by the "Louisiana Slave Database" data collected by Professor Gwendolyn Hall and her research team. The first is that manumitted slaves are more likely to have come from smaller plantations. The second is that women, one of whose tasks of raising children is quite difficult to monitor, are more likely to be manumitted. The data suggest that skilled workers are significantly less likely to be manumitted than unskilled workers, a fact that stands in contrast to Stefano Fenoaltea's model of incentives in slavery but is fully consistent with the model presented here.

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1 Introduction

In his 1984 paper, Stefano Fenoaltea develops a transaction-costs based model in hopes of providing a coherent story about which slaves were manumitted; that is, which slaves were either set free by their masters or were able to purchase their freedom. (Fenoaltea 1984) His model relies primarily on the notion that there are differential effects of providing positive and negative incentives for captive workers. Negative incentives, such as threats to take away comforts or the act of physical punishment, he believes, elicit more effort but less care. A field-hand facing the constant threat of physical punishment will work harder because of this anxiety. However, she will work with less care than she would in the absence of such threats either because of stress or out of spite.

Positive incentives, on the other hand, elicit goodwill on the part of the slaves, and the absence of threats leaves the slave in a more calm state of mind, allowing for more careful work. Positive incentives potentially include material rewards, money, and as Christopher Morris describes, permission to participate in what he refers to as the internal economy. (Morris 1998) This internal economy allowed slaves to use idle land to produce tradeable commodities, giving them the opportunity to earn as much as $50 per year. During the time period considered in this paper, the price of a slave was less than a thousand dollars, and while there was a premium associated with purchasing one's freedom (Cole Forthcoming), a slave who performed a task which was associated with positive incentives could potentially be able to do so.

Because positive incentives would be more effective in eliciting care, Fenoaltea's model has two empirical predictions. The first is that those slaves who performed relatively autonomous tasks, such as herding or housekeeping, that do not involve being monitored continuously would be more likely to be manumitted. Secondly, it suggests that those slaves involved with tasks requiring the exercise of care would also be more likely to be manumitted. There is a "tendency of skilled slaves to be motivated by rewards (and be manumitted)," according to Fenoaltea. Using a detailed data set on manumission and slavery, I show that this latter prediction is shown to be false, at least in Louisiana during the period between 1719 and 1820.

Furthermore, Fenoaltea's model relies on the psychological assumption that negative incentives work well in encouraging hard work and that they are simultaneously prohibitive in getting workers to work carefully. In addition to citing several older psychological studies, he describes this "deleterious effect of excessive stiumlation in skilled tasks" as "familiar to anyone who has seen students, or athletes, perform poorly in pressure situations." While this is certainly a vivid example, it is not consistent with the idea that some people actually perform quite well on certain tasks under pressure (think Assistant Professors), which has been confirmed by recent research. (Markman, et. al. 2006)

I sidestep these psychological issues and develop a simple contract theoretic model of incentives that captures some of the same empirical predictions about the importance of the nature of tasks in predicting who is more likely to be manumitted. In addition, my model performs at least weakly better in one dimension, in that it does not predict that skilled slaves are more likely to be manumitted, and it also yields a prediction about the importance of the size of the estate on manumission rates. Section 2 develops the model and highlights a key comparative static. In section 3, I take the model to the data and show that it is consistent with several observed facts. Section 4 concludes.

2 The Model

In this section, I construct a modified version of the simple principal-agent model that has three key features. The first feature is that, by replacing the standard individual rationality constraint, which captures the idea that, due to an agent's outside options, the principal is limited in the extent to which he can take advantage of the agent, with a survival constraint, which instead is designed to underscore the concept that punishments cannot be so severe as to kill the agent. That is, a master is only able to beat a slave so much before having to bear prohibitive costs of medical care or the loss of the slave.

Payoffs are defined relative to an endogenous living standard. Payoffs in excess of this living standard are considered to be rewards, to which I occasionally refer as "positive incentives", and payoffs below such a living standard are punishments, or negative incentives. I assume that negative incentives are costless to provide up to a point, beyond which they are infinitely costly. A more regular convex punishment cost function is likely to produce the same results, but at the cost of a less analytically tractable problem. It makes sense to think of punishment costs as being convex in the sense that the more physical harm that is done to a slave, the more likely he/she is to need expensive medical care.

In this model, I distinguish between tasks that are easy to monitor and those that are more difficult to monitor in a simplistic, intuitive manner. In order to induce high levels of effort on the part of the agent, the difference between the payoffs when output is high and when output is low must be made sufficiently large. The performance in a task that is more difficult to monitor is evaluated less frequently, and thus in order to induce high levels of effort for multiple periods, this difference must be made larger. Due to the convexity of the punishment cost function and the linearity in the reward cost function (it costs one dollar to provide a one dollar reward), an asymmetry arises, leading to the use of positive incentives only when this difference must be made sufficiently large.

Finally, I endogenize the choice of living standard. By providing a higher living standard to his slaves, a master is better able to induce high levels of effort using less costly negative incentives. Consistent with the idea that slaves were often housed together in Louisiana (Morris, 1998), I model the costs of providing better living standards as exhibiting increasing returns to scale with respect to the number of slaves (on a per slave basis, it is cheaper to house all slaves in a single large room) and decreasing returns with respect to the level of the living standards offered. (Gourmet meals prepared for hundreds of slaves are likely to be quite expensive!)

2.1 Punishment and Reward

Suppose there is a risk neutral principal P and N risk averse agents $A_1, \ldots, A_N$. There are two possible actions that agent i can take $a \in \{a_L, a_H\}$ at costs c_{Li} and c_{Hi} respectively, with $c_{Li} < c_{Hi}$, and there are two levels of output $y \in \left\{y^L, y^H\right\}$ that can result. Under action a, the probability of output level y is denoted by p_a^y. Suppose agents differ in the level U_{0i}, where U_{0i} is defined to be the value such that if the agent ever receives a utility level $u_i < U_{0i}$ in a period, then he/she no longer produces any output in any future period. This can be interpreted as either severe injury or death.

The principal mandates a level w_{0i} for each agent, which can be interpreted as the "living standard" of agent i. That is, w_{0i} is a measure of the amenities that agent i is granted in his/her daily life. Consider a contract in which the principal offers a wage $w^H - w_{0i}$ when output is y^H and enacts a punishment equal to $-\left(w^L - w_{0i}\right)$ when output is y^L, where I will show that $w^L < 0$. Suppose punishments are costless to the principal.

For simplicity, I will assume that the agents' utility functions are given by $u_i = v(t) - c_i = t^\gamma - c_i$, where t is either equal to $w^H - w_{0i}$ or $w^L - w_{0i}$, and $c_i \in \{c_{Li}, c_{Hi}\}$ represents the cost of effort. For now, let us consider the contract between a single agent and the principal. In order to induce agent i to take action a_H at the lowest cost possible, the principal wants to

$$\min_{w^L, w^H} \left(w^H - w_{0i}\right) p_H^H$$

subject to

$$p_H^H \left[\left(w^H - w_{0i}\right)^\gamma - c_{Hi}\right] + p_H^L \left[\left(w^L - w_{0i}\right)^\gamma - c_{Hi}\right] \geq \tag{IC}$$
$$p_L^H \left[\left(w^H - w_{0i}\right)^\gamma - c_{Li}\right] + p_L^L \left[\left(w^L - w_{0i}\right)^\gamma - c_{Li}\right]$$

$$\left(w^L - w_{0i}\right)^\gamma \geq u_{0i} + c_{Hi} \equiv U_{0i}. \tag{SC}$$

Note that (SC), the survival constraint is not the same as the individual rationality constraint that is standard in such a problem. An individual rationality constraint ensures that an individual does not want to take his or her outside option over a certain contract. Since slaves had no such option, no such constraint is present. However, it must be the case that punishments for low output are not so high as to kill or seriously injure a slave, and this is the motivation behind the (SC) constraint.

For simplicity, I will solve this problem for the risk neutral case when $\gamma = 1$. Risk neutrality on the part of the agents is problematic in contract theory, but it is used here only for simplicity and can be thought of as the limiting case as $\gamma \downarrow 1$, which I show in the appendix. The solution is given by

$$\begin{aligned} w^H &= U_{0i} + w_{0i} + \frac{c_{Hi} - c_{Li}}{p_H^H - p_L^H} \\ w^L &= U_{0i} + w_{0i} \end{aligned}$$

In particular, we see that $w^H - w^L = \frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}$. If $-w_{0i} > U_{0i} > -\frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}$, then we have that $w^H > 0$ and $w^L < 0$. A very important necessary condition for this is that $w_{0i} < \frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}$. If it is the case that $w_{0i} \geq \frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}$, that is, if living standards are sufficiently hospitable, and if $U_{0i} < -w_{0i}$, we will have that $w^H = 0$ and $w^L < 0$. That is, the threat of taking away decent living standards alone can induce high levels of effort, provided that the slave is not so sickly as to need high living standards in order to survive.

2.2 Monitoring

The notion of monitoring has not been well embedded into the contract theory literature, though there are numerous early references to it (Jensen, Meckling 1976) and it is clearly important in developing a workable incentive scheme. Here, I develop a quite primitive approach to monitoring, which is simple enough to allow for tractability, but rich enough to provide some empirical implications.

Consider a model of repeated contracting between the principal and the agents in which the principal has all the bargaining power. Suppose each agent is assigned a particular task that differs in its ability to be monitored. Further, suppose there is an easily monitored task and a task which is more difficult to monitor. The performance on a task that is easily monitored is evaluated every T_1 periods, and the appropriate rewards and punishments are apportioned. For tasks that are more difficult to monitor, the performance is evaluated every T_2 periods, where $T_2 > T_1$.

Each agent i must choose an action each period, and I assume that costs of effort are linear in the length of time incurred. That is, if agent i chooses action a_H for T_1 periods, she incurs the cost $T_1 c_{Hi}$. Provided that the principal wants to induce the high level of effort on the part of all the agents, we have that, for the easily monitored task, if $U_{0i} > -T_1 \frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}$, then it must be the case that $w^H > 0$. For the task which is more difficult to monitor, we need only that $U_{0i} > -T_2 \frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}$ in order to ensure that $w^H > 0$. Since $T_2 > T_1$, it is necessarily the case that $1\left\{U_{0i} > -T_2 \frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}\right\} \geq 1\left\{U_{0i} > -T_1 \frac{c_{Hi}-c_{Li}}{p_H^H - p_L^H}\right\}$, and thus it is more likely to be necessary to pay a positive wage for high output in the task that is more difficult to monitor than in the task that is easier to monitor.

2.3 Living Standards

In this section, I allow for the principal to choose w_{0i}. Giving the principal this control over the choice of w_{0i} is tantamount to giving the principal control over whether rewards or punishments will be used. For simplicity, I will assume that $w_{0i} = w_0$ for all i, though a richer model would allow for different standards of living across individuals, since doing so would allow for different incentives to be used for different individuals, as in some tasks, punishment with high living standards might be more cost effective than reward with low living standards, while the opposite might be true in other tasks.

Let us focus on the overall cost to the principal of providing living standards in a given period. At time t, then the principal pays $\sum_{i=1}^{N} w_0 = N w_0$. Suppose there is a convex housing technology, which can provide housing of quality w_0 to N agents at a cost of $\alpha(N) w_0^{\delta}$, where $\delta > 1$ and $\frac{d}{dN}\alpha(N) < 1$. This latter term is designed to capture the initial increasing returns to scale of providing housing. The principal then wants to

$$\max_{w_0} \left\{ N w_0 - \alpha(N) w_0^{\delta} \right\}.$$

The solution to this problem is simply $w_0^* = \left(\frac{N}{\alpha(N)\gamma}\right)^{\frac{1}{\delta-1}}$, which is increasing in N.

3 Empirical Evidence

3.1 Data

The primary data set I use in this paper is the "Louisiana Slave Database," and to a lesser extent, I draw some information from the "Louisiana Free Database."[2] These data sets were constructed by Gwendolyn Hall, a professor of history at Rutgers University, who spent many years examining over a hundred thousand documents related to slavery in Louisiana. The "Louisiana Slave Database" consists of one hundred thirty variables drawn from the documents describing characteristics of the individual slaves (age, sex, character, skills, origin, family, whether or not the slave was involved in a revolt, etc.), limited information about the estate to which the slaves belonged, and information about the sale of the slaves and whether or not they were set free. The "Louisiana Free Database" contains similar information about those slaves which were documented as having been set free, and it also paints a picture about the person who freed the slave free as well as some limited information about the conditions of and reasons for the manumission. Figure 1 shows an example of a document on which these data sets are based.

Because the two data sets do not uniquely identify the slaves in a way that makes it possible to compare the characteristics of the slaves listed in the "Louisiana Free Database" to those slaves listed in the "Louisiana Slave Database," I focus primarily on the latter data set to draw inferences about what types of slaves are manumitted. I use the former data set only to help enrich the picture of manumission when appropriate. Table 1 lists some descriptive statistics about the variables from the "Louisiana Slave Database" that I use in the next subsection.

[2] Both are available online at http://ibiblio.org/laslave/downloads.php

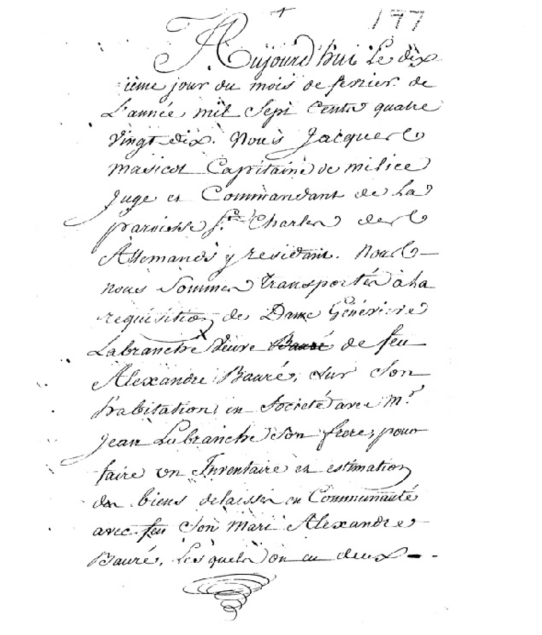
177
Aujourd'hui le dix-
ième jour du mois de janvier de
l'année mil sept cent quatre
vingt dix. Nous Jacques
Masicot Capitaine de milice
Juge et Commandant de la
paroisse St Charles des
Allemands y residant. Nous
nous sommes transporté à la
requisition de Dame Geneviève
Labranche Veuve de feu
Alexandre Baure, sur son
habitation en société avec Mr
Jean Labranche son frère pour
faire un inventaire et estimation
des biens delaissés en communauté
avec feu son mari Alexandre
Baure, lesquels on a

Figure 1. Slave Inventory Sheet. http://ibiblio.org/laslave/inventory.php

Table 1: "Louisiana Slave Database"

$(N = 100,666)$	Number	Percentage
Age is coded	$73,979$	73.5
Sex is coded	$90,610$	90.0
Skill is indicated	$7,119$	7.1
Slave is listed as expert	$1,191$	1.2
Character category is coded	855	0.8
Sickness is indicated	$1,761$	1.7
Slave is listed as "freed"	$4,340$	4.3

Examples of skills that are listed in the data set include "cook," "herdsman," "seamstress." Character categories include "thief," "behavior problems," "strong," and "good character."

3.2 Findings

The model presented in this paper has two empirical predictions regarding which types of slaves one would expect to be able to purchase freedom. The first is that slaves from smaller estates are more likely to be manumitted than slaves from larger estates. This result is driven by the notion that, since punishment (the stick) is cheaper for the principal to carry out than reward (the carrot), it may be in the principal's best interest to allow for a wider range of punishments by increasing the living standards of slaves, giving them, in some sense, "more to lose." Because on a per slave basis, it is cheaper to provide housing and amenities for more slaves than for fewer, this model suggests that larger estates will be more effective at punishing those who do not exhibit the high level of effort than smaller estates. Since, in this model, punishment is a substitute for reward, it would then follow that larger estates do not need need to reward slaves for good effort as much as smaller estates do, and hence manumission rates for slaves at larger estates will tend to be lower than for slaves at smaller estates.

The number of slaves at a given estate is only listed for those estates for which a document containing a complete list of the slaves was found by Professor Hall and her team. Only about three percent of the entries in the data set contain this information, so any conclusions drawn by using this data must be taken with a grain of salt. The light gray line of Figure 2 represents the CDF of estate size in the data set, with attention restricted, of course, to those estates for which data on size are available. The black line plots the CDF of estate size of the estates for which the slave listed in the data set has been manumitted. For example, 67% of estates had ten or fewer slaves, while restricting our attention to those estates which had manumitted a slave, 82% had fewer than ten slaves. From this figure, we see that manumitted slaves came disproportionately from smaller estates.

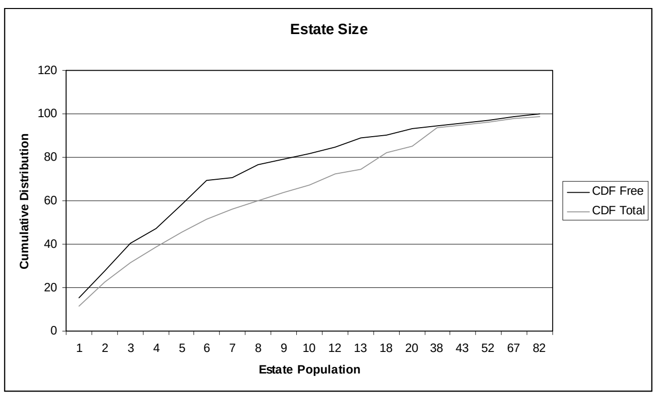

Figure 2

During the period from which this data set is drawn, Louisiana primarily produced sugar and cotton and, to a lesser extent, tobacco. Sugar and cotton plantations were very well-suited to the "gang labor system" in which "field slaves usually worked in gangs of 10 to 20 hands, each of which was headed by a 'driver' - a slave who, with whip in hand, pushed his gang to achieve the assigned task." (Fogel 1989, p. 26) The gang labor system provided the master with a great way of monitoring the progress of his slaves. Thus, according to this model, those slaves who worked in gangs as field hands would be less likely to receive positive rewards and hence would be less likely to be able to purchase their freedom. Unfortunately, the data sets contain very little information about the daily tasks the individual slaves performed, so it is not possible to see if this prediction is borne out.

In examining who was manumitted in the data set, one striking feature emerges. The "Louisiana Slave Database" suggests that women are almost two and a half times as likely to be manumitted as men while according to the "Louisiana Free Database," they are only eleven percent more likely to be freed without having to purchase their freedom. Thus, more women than men purchased their freedom. However, according to Fogel, roughly 75% of adult male slaves and 83% of adult female slaves participated in the gang labor system. This presents a bit of a puzzle for the model, since it would predict that men would be slightly more likely to be manumitted than women.

One possible explanation is that, while the daily tasks that women performed were quite easily monitored, women also had the chore of raising their children. The quality of the attention that the mother gave to her child could not be as easily monitored as other tasks. "Masters and slave mothers argued over how to care for children." (Morris 1998, p. 989) Thus, a reward system was potentially important in ensuring that they raise their children in a way that would be in the best interest of their master. Morris provides some evidence that women were rewarded for having children. "Often, slave owners held out extra rations of food and clothing to women as incentives to have children."

Because there is no information about which type of task each slave performed, the only other piece of information contained within the data set which could potentially allow one to evaluate whether or not slaves performing tasks which are more difficult to monitor are more likely to be set free is the information about the skills that a slave has. In contrast to Fenoaltea's transaction-costs model, the model presented in this paper is agnostic regarding the notion that skilled slaves are more or less likely to be manumitted than non-skilled slaves, as there are two effects going in opposite directions. The first is that a skilled slave generally will perform a task which is more difficult to monitor, and hence one would expect skilled slaves to be more likely to be manumitted. The opposing effect is that skilled slaves are likely to be better at what they do than non-skilled slaves, and hence the costs of effort may indeed be lower, so it may be easier for the principal to induce high levels of effort.

Though noisy, the data suggest that this latter effect is the more dominant. Simply comparing means, skilled slaves are 54.6% as likely as non-skilled slaves to be manumitted eventually. Furthermore, skilled slaves who are listed as "experts" at their task are 40.6% as likely to be manumitted as slaves who are not considered to be experts. If we interpret an expert as an individual for whom exerting a high level of effort is less costly, then this is fully consistent with the model.

In order to test the statistical significance of each of these effects, I use a logit model to determine the effects on the probability of being freed on several dummy variables. $FEMALE_i$ is a dummy variable equal to one if the slave is a female, $SKILLED_i$ is equal to one if a skill is associated with slave i, and $EXPERT_i$ is one if slave i is reported to be an expert. To check the robustness of my results, I include three control varibles. $SICK_i$ is equal to one if slave i is listed as having a disability of a mentall illness, $CHARACTER_i$ takes on the value of one if the slave is listed as being strong, intelligent, or having a good character overall. AGE_i is the age of the slave at the time the document was written. Observations for which no gender or no age is reported are dropped. In order to ensure that the assumptions about the error terms are not driving the results, I also run a probit as well as a linear probability model.

Table 2: Probability of Manumission

	Logit		Probit		LPM	
	I	II	I	II	I	II
Female	0.0366*** (0.0017)	0.0367*** (0.0017)	0.0370*** (0.0017)	0.0370*** (0.0017)	0.0375*** (0.0017)	0.0375*** (0.0017)
Skilled	−0.0273*** (0.0024)	−0.0278*** (0.0023)	−0.0278*** (0.0024)	−0.0284*** (0.0023)	−0.0262*** (0.0023)	−0.0269*** (0.0023)
Expert	−0.0155** (0.0078)	−0.0175** (0.0076)	−0.0142* (0.0077)	−0.0159** (0.0076)	−0.0090** (0.0045)	−0.0109** (0.0047)
Character		0.0342** (0.0173)		0.0336** (0.0164)		0.0226* (0.0116)
Sick		0.0078 (0.0067)		0.0075 (0.0067)		0.0073 (0.0060)
Age		−0.00001 (0.00006)		−0.00003 (0.00006)		−0.00001 (0.00006)
Constant					0.0399*** (0.0010)	0.0401*** (0.0018)
N	73026	73026	73026	73026	73026	73026

*** = coefficient significantly different from zero at the 1-percent level

** = coefficient significantly different from zero at the 5-percent level

* = coefficient significantly different from zero at the 10-percent level

The results of these regressions are listed in Table 2. For the Logit and Probit models, I list the marginal effects in order to ensure that the coefficients across the six specifications are comparable. I find statistical evidence at the 1-percent level in support of the idea that females are more likely to be manumitted and that skilled slaves are less likely to be manumitted. I also find evidence at the 5- and 10- percent level that expert slaves are less likely to be manumitted. The character of a slave, which is beyond the scope of my model, is also found to be statistically significant, while whether a slave had a disability or chronic illness is not found to be of importance, and neither is age.

4 Conclusion

The model presented in this paper does at least as well as Fenoaltea's model in predicting that those slaves who perform tasks that are not easily monitored will have a greater tendency to be manumitted, though the data do not allow me to test this hypothesis. In addition, it performs better in that it does not suggest necessarily that skilled slaves are more likely to be manumitted, a prediction from Fenoaltea's model that is contradicted by the data presented here. This model also yields another useful prediction regarding the size of the estates from which manumitted slaves come. That is, a manumitted slave is more likely to come from a smaller estate than from a larger one.

Two aspects of this model can can potentially be of use in explaining the differences in the low manumission rates in the American South and the much higher manumission rates of Ancient Rome. Temin states that "frequent manumission was a distinguishing feature of Roman slavery; slaves in the early Roman empire could anticipate freedom if they worked hard... or accumulated a peculium with which to purchase it." (Temin 2004) First, the slaves in Roman times performed tasks that were not easily monitored. They were often sent to run errands in town for their masters, or they had managerial jobs "like a *vilicus*" (ibid). Secondly, the idea that masters who owned more slaves were less likely to free any of them, could also explain higher manumission rates in Rome if the average master owned few slaves.

Finally, since the provision of higher living standards in this model is necessary only to induce high levels of effort, a master acting according to this model would have very little incentive to provide decent living standards for those who are not productive. Steckel writes about the malnutrition of child slaves in 19th century America. (Steckel 1986) Children were often so malnourished that by age 6, they were on average in the first percentile of height for children their age. Upon reaching working age, they climbed to higher percentiles, clearly indicating that higher living standards were provided for those who needed them for incentive reasons. The institution of slavery is no doubt brutal, but nevertheless, it is still subject to many of the same pitfalls of modern economic organization. Incentives must be provided, and the economist, by tracing out these incentives can perhaps provide some insights into its structure.

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Appendix

Proposition 1 *For the model presented in section 2, when $\gamma > 1$, the following expression holds*

$$w^H - w^L \propto \frac{c_H - c_L}{p_H^H - p_L^H}$$

if we assume that $-w_{0i} > U_{0i} > -\frac{1}{p_H^H - p_L^H}(c_H - c_L)$.

Proof. Since punishments are costless, it is rational for the principal to punish as much as he can upon receiving a low quality output in order to make (IC) more slack. This ensures that (SC) will hold with equality. (IC) will also hold with equality, since otherwise, the principal could decrease w^H and still induce action a_H. This gives us

$$p_H^H\left[\left(w^H - w_{0i}\right)^\gamma - c_H\right] + p_H^L\left[\left(w^L - w_{0i}\right)^\gamma - c_H\right] = p_L^H\left[\left(w^H - w_{0i}\right)^\gamma - c_L\right] + p_L^L\left[\left(w^L - w_{0i}\right)^\gamma - c_L\right],$$

which, since $p_H^H = 1 - p_H^L$ and $p_L^H = 1 - p_L^L$, we can rearrange to obtain the following expression

$$\left(w^H - w_{0i}\right)^\gamma - \left(w^L - w_{0i}\right)^\gamma = \frac{c_H - c_L}{p_H^H - p_L^H}.$$

By a simple first order Taylor expansion, we then have that

$$\begin{aligned}\frac{c_H - c_L}{p_H^H - p_L^H} &\approx (w_{0i})^\gamma - (w_{0i})^\gamma + \begin{bmatrix} w^H & w^L & -2w_{0i}\end{bmatrix}\begin{bmatrix}\gamma (w_{0i})^{\gamma-1} \\ -\gamma (w_{0i})^{\gamma-1} \\ 0\end{bmatrix} \\ &= \left(w^H - w^L\right)\gamma (w_{0i})^{\gamma-1},\end{aligned}$$

and, provided that $w_{0i} > 0$ (if necessary, an expansion around $-w_{0i}$ ensures that this holds for the mathematical possibility that $w_{0i} < 0$), we have

$$w^H - w^L \approx \frac{1}{\gamma (w_{0i})^{\gamma-1}}\frac{c_H - c_L}{p_H^H - p_L^H} \propto \frac{c_H - c_L}{p_H^H - p_L^H},$$

which is the desired result. Further, in the limit as $\gamma \to 1$, we have that $w^H - w^L \approx \frac{c_H - c_L}{p_H^H - p_L^H}$. ■