

is awesome!

Table 1. Definitions of  $k$ -class estimators.

$k$ -class estimator	$\frac{x'Py - \kappa x'My}{x'Px - \kappa x'Mx}$
LIML	$\kappa = \phi$
F(1)	$\kappa = \phi - \frac{1}{n-K}$
F(4)	$\kappa = \phi - \frac{4}{n-K}$
F(opt)	$\kappa = \phi - \frac{3+1/\rho^2}{n-K}$
Nagar	$\kappa = \frac{K-2}{n} / (1 - \frac{K-2}{n})$
2SLS	$\kappa = 0$

$\phi$  is equal to the smallest eigenvalue of the matrix  $W'PW(W'MW)^{-1}$ , where  $W \equiv [y, x]$ .

number of observations  $n$ ,  $\rho$  and the (theoretical)  $R^2$  of the first-stage regression.<sup>6</sup> Using the normalization, the often used concentration parameter approach yields  $\delta^2 \approx nR^2/(1 - R^2)$ .

The estimators that we consider are

- LIML—see e.g. Hausman (1983) for a derivation and analysis. LIML is known not to have finite sample moments of any order. LIML is also known to be median unbiased to second order and to be admissible for median unbiased estimators (see Rothenberg 1983). The higher-order mean bias for LIML does not depend on  $K$ .
- 2SLS—the most widely used IV estimator. 2SLS has finite sample bias that depends on the number of instruments used  $K$  and inversely on the  $R^2$  of the first-stage regression (see e.g. Hahn and Hausman 2002b). The higher-order mean bias of 2SLS is proportional to  $K$ . However, 2SLS can have smaller higher-order mean square error (MSE) than LIML using second-order approximations when the number of instruments is not too large (see Bekker 1994; Donald and Newey 2001).
- Nagar—mean unbiased up to second order. For a simplified derivation see Hahn and Hausman (2002b). The Nagar estimator does not have moments of any order.
- Fuller (1977)—this estimator is an adaptation of LIML designed to have finite sample moments. We consider three different estimators with the  $a$  parameter in (4) chosen to take on values 1 or 4 or the value that minimizes higher-order MSE. The optimal estimator uses  $a = 3 + 1/\rho^2$ . This choice minimizes the higher-order MSE regarded as a function of  $a$ . These three estimators will be abbreviated F(1), F(4) and F(opt) throughout the rest of the paper. For the optimal Fuller estimator, the higher-order bias is greater, but the MSE is smaller. This last estimator is infeasible since  $\rho$  is unknown in an actual situation, but we explore it for completeness. The optimal estimator has the same higher-order MSE as LIML up to  $O(\frac{K}{n^2})$  but unlike LIML also has existing finite sample moments.
- JN2SLS—the higher-order mean bias does not depend on  $K$ , the number of instruments. JN2SLS has finite sample moments. However, as we discuss later, its MSE exceeds the other estimators in some situations.
- JIVE—the Jackknifed IV estimator of Phillips and Hale (1977) and Angrist *et al.* (1999). This estimator is higher-order mean unbiased similar to Nagar, but we conjecture that it does not have finite sample moments. The Monte Carlo results demonstrate a likely absence of finite sample moments, which is further substantiated by Davidson and MacKinnon (2004).
- OLS—this estimator is to be considered as a benchmark.

<sup>6</sup>The theoretical  $R^2$  is defined later in (5).