

Answer two questions (50 points each).

1. Consider the model under the usual assumptions:

$$y_1 = \beta_{11} y_2 + \gamma_{11} z_1 + \gamma_{12} z_2 + \gamma_{13} z_3 + \epsilon_1$$

$$y_2 = \beta_{21} y_1 + \gamma_{21} z_1 + \epsilon_2$$

- (i) Discuss the rank and order conditions for identification.
  - (ii) Write down the reduced form and discuss how you can estimate the  $\beta$ 's and  $\gamma$ 's from the reduced form estimation.
  - (iii) For the second equation you test the hypothesis  $H_0 : \gamma_{21} = 0$  and do not reject. What do you conclude about the identification of the first equation.
  - (iv) You perform tests of overidentification for both equations (ignore part (iii) above). Give the form of the tests. What do you expect in terms of results? What do you conclude if you reject for either equation?
2. You have a limited information model:

$$y_1 = Y_1 \beta_1 + Z_1 \gamma_1 + \epsilon_1$$

- (i) Specify the 2SLS estimator and LIML estimator for the equation. How do you expect their performance to compare?
  - (ii) You expect you may have conditional heteroscedastity. Specify the efficient IV (EIV) estimator (or GMM estimator) in this situation.
  - (iii) You notice that the EIV estimates looks much different than the 2SLS estimates? What do you conclude? Can you give a test for your conclusion?
  - (iv) Specify the test of overidentifying restrictions for the EIV estimator. Suppose you do not reject using this test but you do reject the overidentifying restrictions using the 2SLS estimator. What do you conclude?
3. You have the model

$$y_1 = \beta y_2 + \epsilon$$

$$y_2 = Z\pi + v$$

where  $\sigma_{\epsilon v} \neq 0$ .

- (i) What is the second-order bias of the 2SLS estimator for  $\beta$ ? What happens if you have only 1 instrument?
- (ii) Suppose  $\text{plim} \frac{1}{N} Z' \epsilon \neq 0$ . Derive the second-order bias of 2SLS.
- (iii) Derive the bias of LIML or Fuller under the condition in (ii). Do LIML or Fuller fix the problem?
- (iv) Consider the situation in (ii). Compare using OLS in this situation to IV where you do not have a valid instrument.