Problem set will be available today

FIML and LIML

$$YB + Z\Gamma = U$$

$$\mathcal{L} = c + T\log|\det B'| + \frac{T}{2}\log|\det \Sigma^{-1}| - \frac{1}{2}tr(\Sigma^{-1}u'u)$$

$$(1)\quad \frac{\partial \mathcal{L}}{\partial B^u} = T\frac{\partial \log|\det B'|}{\partial B^u} - \frac{1}{2}\frac{\partial tr(\Sigma^{-1}u'u)}{\partial B^u} = 0$$

Recall: Magnus and Neudecker

- Matrix algebra for Econometrics

- $\frac{\partial \log|\det A|}{\partial A} = (A')^{-1}$
- $\frac{\partial tr(AB)}{\partial A} = B$

$$\Rightarrow (1) = T(B^{-1})' - Y'U\Sigma^{-1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Gamma^u} = -\frac{1}{2}\frac{\partial tr(\Sigma^{-1}u'u)}{\partial \Gamma^u} = 0$$

$$\Rightarrow -Z'U\Sigma^{-1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Sigma^{-1}} = \frac{T}{2}\Sigma - \frac{1}{2}u'u = 0 \Rightarrow \Sigma = \frac{u'u}{T}$$

$$(1) \Rightarrow 0 = T(B^{-1})' \frac{u'u}{T} \Sigma^{-1} - Y' u \Sigma^{-1}$$

$$= (B^{-1})' (YB + Z\Gamma)' u \Sigma^{-1} - Y' u \Sigma^{-1}$$

$$= (B^{-1})' (B'Y' + \Gamma' Z') u \Sigma^{-1} - Y' u \Sigma^{-1}$$

$$= [Y' + (B^{-1})' \Gamma' Z' - Y'] u \Sigma^{-1}$$

$$= -\Pi' Z' u \Sigma^{-1}$$

$$\left.\begin{aligned} &\Rightarrow [\Pi' Z' u \Sigma^{-1}]^{u} = 0 \\ (2) &\Rightarrow [Z' u \Sigma^{-1}]^{u} = 0 \end{aligned}\right\} \quad \begin{aligned} &[\hat{Y}' u \Sigma^{-1}]^{u} = 0 \\ &[Z' u \Sigma^{-1}]^{u} = 0 \end{aligned}$$

Thus, (clearly) $\bar{D}(\Sigma^{-1} \otimes Z')(\bar{X} \hat{\delta}_{FIML} - \bar{y}) = 0$

$$\Rightarrow \hat{\delta}_{FIML} = (\bar{D}(\Sigma^{-1} \otimes Z')\bar{X})^{-1} (\bar{D}(\Sigma^{-1} \otimes Z')\bar{y})$$

cf $\hat{\delta}_{3SLS} = (\bar{X}'(\Sigma^{-1} \otimes P_Z)\bar{X})^{-1} (\bar{X}'(\Sigma^{-1} \otimes P_Z)\bar{y})$

$$\sqrt{T}(\hat{\delta}_{FIML} - \hat{\delta}_{3SLS}) \xrightarrow{P} 0.$$

Anderson and Rubin

Let $\hat{\lambda} = \min \dfrac{(Y - X\beta)' P_Z (Y - X\beta)}{(Y - X\beta)'(Y - X\beta)}$

$$\hat{\beta}_{LIML} = (X' P_Z X - \hat{\lambda} X'X)^{-1} (X' P_Z Y - \hat{\lambda} X'Y)$$

Theil: $K = 1 + \lambda$

Michael Powell-Fa06 Date 10/13/06 Course 14.383 Page 3

$\hat{\beta}_{LIML}$ is median unbiased

Fuller: $K = 1 + \hat{\lambda} - \frac{1}{T}$

Hypothesis Testing

Most important question in SEM: $\text{plim } \frac{Z'\varepsilon}{T} = 0$?

Limited information:

Test 1: $\hat{\varepsilon}_{2SLS} = Z\alpha + \eta$ $\quad H_0: \alpha = 0$

- This is just an F-test.
- If $K=1$, then $Z'\hat{\varepsilon} = 0$ by construction,
  - $Z'(Y - X\hat{\beta}) = Z'Y - Z'X(X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'y$

$$= Z'Y - \underbrace{Z'X}_{\text{scalar}}(Z'X)^{-1}\underbrace{(Z'Z)}_{\text{scalar}}(X'Z)^{-1}X'Z(Z'Z)^{-1}Z'Y$$

(Sargan test)
(Omnibus test)

$$= Z'Y - Z'Y = 0$$

Test 2: 2SLS: $Q = \frac{\hat{\varepsilon}' P_Z \hat{\varepsilon}}{\hat{\sigma}^2} \xrightarrow{d} \chi^2_{(K-k_1)}$ is the objective function.

- If regression is correct, we want $Q \approx 0$.
- $Q = TR^2$ from regressing $\hat{\varepsilon}_{2SLS} = \beta + Z\alpha + \eta$

alternatively, we can use OIV.

$$Q = (Z'\hat{\varepsilon})' \hat{V}^{-1} (Z'\hat{\varepsilon}) \xrightarrow{d} \chi^2(K-k_1)$$

- Both of these are biased towards rejection.

Test 3: Divide Z's into two groups:
(Hausman)
- Z_1 - sure about those
- Z_2 - not sure about.
- H_0: $\text{plim} \frac{Z_2' \varepsilon}{T} = 0$, H_1: $\text{plim} \frac{Z_2' \varepsilon}{T} \neq 0$
- Construct $\hat{\beta}_{2SLS}(Z_1, Z_2)$ and $\hat{\beta}_{2SLS}(Z_1)$
  - Under H_0: $\hat{\beta}_{2SLS}(Z_1, Z_2)$ and $\hat{\beta}_{2SLS}(Z_1)$ are both consistent and $\hat{\beta}_{2SLS}(Z_1, Z_2)$ is more efficient
  - Under H_1: $\hat{\beta}_{2SLS}(Z_1, Z_2)$ is inconsistent and $\hat{\beta}_{2SLS}(Z_1)$ is consistent.
- Construct the usual Hausman test

Test 4: Test based on the reduced form coefficients.
- $\frac{\Pi_{12}}{\Pi_{22}} = \frac{\Pi_{13}}{\Pi_{23}}$ nonlinear H_0: $\underbrace{\frac{\Pi_{12}}{\Pi_{22}} - \frac{\Pi_{13}}{\Pi_{23}}}_{h(\Pi)} = 0$
- Wald test $h(\hat{\Pi})' [H \Psi H']^{-1} h(\hat{\Pi}) \sim \chi^2$

Test 5: Under heteroskedasticity, can do Hausman test on 2SLS vs OIV.
- Under H_1, both inconsistent, but $\text{plim} \hat{\beta}_{2SLS} \neq \text{plim} \hat{\beta}_{OIV}$

Test on the system:

1. Hausman test: $\hat{\beta}_{3SLS}$ vs $\hat{\beta}_{2SLS}$ eqn by eqn.
2. Sargan test: 3SLS objective function:

$$Q = \hat{\varepsilon}'(\hat{\Sigma}^{-1} \otimes P_Z)\hat{\varepsilon} \xrightarrow{P} \chi^2(\sum_{i=1}^{m} \text{deg. of overid})$$

3. FIML: do a likelihood ratio test:
   - $2[\ln(L_U) - \ln(L_R)] \xrightarrow{P} \chi^2$
     - Divide into two sets of instruments
   - alternatively, use Wald or LM
4. Test whether one or more eqns are correctly specified.
   - FIML vs LIML eqn by eqn.
   - Hausman test