Limited Information Estimation - not necessarily efficient
Full Information Estimation - if one equation is misspecified, then the estimators will be inconsistent.

Limited Information:

$$y_1 = \underbrace{Y_1}_{\text{endogenous vars other than } y_1} \beta_1 + Z_1 \gamma_1 + \varepsilon_1 \quad \leftarrow \text{Equation of interest}$$

Let $X_1 = [\underbrace{Y_1}_{\text{endogenous}} \ \underbrace{Z_1}_{\text{exogenous}}]$ - all the rhs, $\delta_1 = \begin{bmatrix} \beta_1 \\ \gamma_1 \end{bmatrix}$

Reduced form: $X_1 = \underbrace{Z}_{\text{all exogenous variables}} \Pi + V_2$, $Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$

Structural for 1
Reduced form for other eqns

$$\Rightarrow \begin{bmatrix} Y_1 \\ Z_1 \end{bmatrix} = Z \begin{bmatrix} \Pi_2 \\ I \end{bmatrix} + V_2$$

IV/2SLS

$$\hat{\delta}_1 = (X_1' P_Z X_1)^{-1} X_1' P_Z y_1$$

$$= (W_1' X_1)^{-1} W_1' y_1 \quad \text{where } W = Z A$$

and $\hat{A} = [\hat{\Pi}_2 \ \ I]$

$\Rightarrow W = [Z\hat{\Pi}_2 \ \ Z]$

IV/2SLS need not be unbiased

$$E[\hat{X}_1' \varepsilon_1] = E[Z\hat{\Pi}_2 \varepsilon_1]$$

$$\hat{\Pi}_2 = (Z'Z)^{-1} Z' X_1$$
$$= \Pi_2 + (Z'Z)^{-1} Z' v_2$$

$$\Rightarrow E[\hat{\Pi}_2] = \Pi_2 + E[(Z'Z)^{-1} Z' v_2] \neq \Pi_2$$

Facts about IV/2SLS

1. Consistent
2. Biased
3. asymptotically Normal
4. $\hat{\sigma}_{11} = \frac{1}{T-K_1} (Y_1 - \underbrace{X_1}_{\text{don't estimate these}} \hat{\delta}_1)'(Y_1 - X_1 \hat{\delta}_1)$
5. R^2 is meaningless
6. Durbin-Watson isn't reliable
7. 2SLS is more efficient than other IV estimators under homoskedasticity.
   - under heteroskedasticity, use OIV.

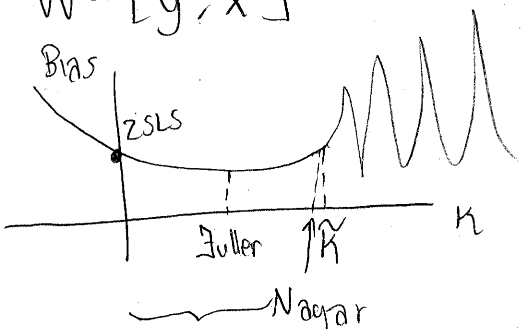
$$y_1 = \delta X_1 + \varepsilon_1$$
$$X_1 = Z \Pi_2 + V_2$$
$$\delta = \frac{X_1' P_Z y_2 - K y_1' Q_Z X_1}{X_1' P_Z X_1 - K X_1' Q_Z X_1}$$

$K = 0 \Rightarrow$ 2SLS

$K = -1 \Rightarrow$ OLS

Theil 1950's:

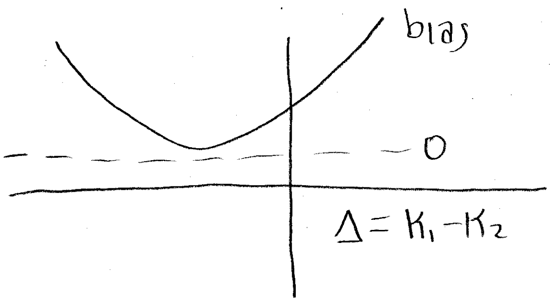
Fuller estimator: $k = \varphi - \frac{1}{n-k}$, where φ is equal to the smallest eigenvalue of $W'PW(W'MW)^{-1}$, where $W = [y, x]$



optimal area: $k \in [0, 0.05]$ approximately

alternatively, consider: $$\delta = \frac{X_1' P_Z y_1 - k_1 y_1' Q_Z X_1}{X_1' P_Z X_1 - k_2 X_1' Q_Z X_1}$$

where k_1 need not equal k_2. (Double k class)



- can get very close to no bias.

Seemingly Unrelated Regression - Multivariate Least Squares

$$\begin{cases} y_1 = X_1 \beta_1 + \varepsilon_1 \\ y_2 = X_2 \beta_2 + \varepsilon_2 \\ \vdots \\ y_N = X_N \beta_N + \varepsilon_N \end{cases}$$

- No endogeneity

Stacking:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} \underline{X}_1 & & \\ & \ddots & \\ & & \underline{X}_N \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_N \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{bmatrix}$$

$$\bar{y} = \bar{\underline{X}} \beta + \bar{\varepsilon}$$

$Var(\varepsilon) = \Sigma \otimes I_T$ ie $E[\varepsilon_{it}\varepsilon_{jt}] = \sigma_{ij} \quad \forall t$

$E[\varepsilon_{it}\varepsilon_{jt'}] = 0 \quad \forall t \neq t'$

Feasible GLS:

$$\hat{\beta}_{FGLS} = (\bar{\underline{X}}'(\hat{\Sigma} \otimes I)^{-1}\bar{\underline{X}})^{-1} \bar{\underline{X}}'(\hat{\Sigma} \otimes I)^{-1}\bar{y}$$

Properties:

1. $\sigma_{ij} = 0 \Rightarrow$ GLS = OLS
2. $\underline{X}_i = \underline{X}_j \Rightarrow$ GLS = OLS
3. $\underline{X}_i \subset \underline{X}_j \Rightarrow$ No efficiency gain in estimating equation j from equation i.
4. $\sigma_{ij} \uparrow\uparrow \Rightarrow$ GLS does very well
5. $\sigma_{\underline{X}_i \underline{X}_j} \downarrow\downarrow \Rightarrow$ GLS does very well
6. If one equation is misspecified, then <u>all</u> are misspecified.
7. Iterate FGLS $\Rightarrow$ get MLE

$\underline{X} = [\underline{Y} \quad \underline{Z}]$ Departing from SUR

$$\hat{\beta}_{3SLS} = (\bar{\underline{X}}'(\underbrace{\hat{\Sigma}^{-1}}_{\text{get these from 2SLS}} \otimes P_{\underline{Z}})\bar{\underline{X}})^{-1} \bar{\underline{X}}'(\hat{\Sigma}^{-1} \otimes P_{\underline{Z}})\underline{Y}$$

FIML

$$\underline{Y}B + \underline{Z}\Gamma = U \qquad u_t \sim N(0, \Sigma)$$

$$u_t \sim f(u_t) \qquad u_t = g(y_t)$$

$$\Rightarrow f_{\underline{Y}}(y_t) = f_u(g(y_t)) \left| \det \underbrace{\left(\frac{\partial g(y)}{\partial y} \right)}_{\text{Jacobian}} \Big|_{y=y_t} \right|$$

$f_u(g(y_t))$ = pdf of u_t evaluated at y_t

$$u_t' = B' y_t' + \Gamma' z_t'$$

$$\Rightarrow \frac{\partial u_t'}{\partial y_t'} = B'$$

$$f_u(y_t) = \frac{1}{(2\pi)^{N/2}(\det \Sigma)^{1/2}} \exp\left\{-\frac{1}{2} u_t \Sigma^{-1} u_t'\right\}$$

$$f_y(y_t) = \frac{1}{(2\pi)^{N/2}(\det \Sigma)^{1/2}} \exp\left\{-\frac{1}{2} u_t \Sigma^{-1} u_t'\right\} |\det(B')|$$

$$\Rightarrow \mathcal{L} = c - \frac{T}{2}\log(\det\Sigma) - \frac{1}{2}\sum_{t=1}^{T}\underbrace{u_t \Sigma^{-1} u_t'}_{\text{scalar}} + T\log\det B'$$

$$= c + \frac{T}{2}\log(\det\Sigma^{-1}) - \frac{1}{2}tr(\Sigma^{-1}u'u) + T\log\det B'$$