BLP property

$$\beta = \arg\min_b E[(y_t - x_t' b)^2]$$

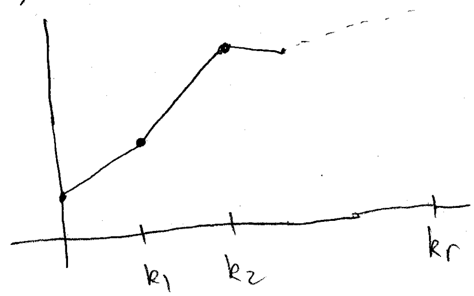
BLA property (Best linear approximation)

$$\beta = \arg\min_b E[(\underbrace{E[y_t | W_t]}_{\text{cond. expect. fcn}} - \underbrace{x_t' b}_{\text{linear approximation}})^2] \qquad x_t = f(W_t)$$

- important to keep this in mind when dealing with misspecification.

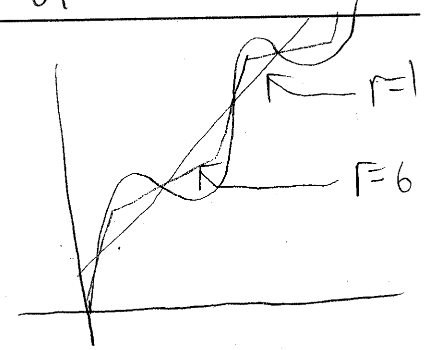
Building Functional Forms:

1. Spline approximation:
   - $w_t \in \mathbb{R}$: eg linear spline series (spline of order 1) with knots $k_1 < k_2 < \dots < k_r$ takes the form
     $x_t = f(w_t) = [1, w_t, (w_t - k_1)_+, \dots, (w_t - k_r)_+]'$, where
     $(a)_+ = 1(a>0) \cdot a.$
   - $w \mapsto f(w)'b$
   - piecewise linear approximation



- eg $f(w)'b = b_0 + b_1 w + b_2 (w - k_1) 1(w - k_1 > 0)$
  - This is continuous at k_1:
    $w \uparrow k_1 \Rightarrow f(k_1^-)'b = b_0 + b_1 k_1$
    $w \downarrow k_1 \Rightarrow f(k_1^+)'b = b_0 + b_1 k_1 + b_2 (k_1 - k_1) 1(w - k_1 > 0)$
    $= b_0 + b_1 k_1$

- <u>Example</u> $g(w_t) = w_t + 2\sin w_t$
  - $w_t \sim \text{uniform}\{1,2,\dots,20\}$



<u>Eg2</u> Cubic splines:
- $x_t = f(w_t) = [1, w_t, w_t^2, w_t^3, (w_t - k_1)_+^3 \cdots (w_t - k_r)_+^3]'$
- $w \mapsto f(w)'b$ is twice differentiable. (check how many continuous derivatives it has.)

2] Power series
- $x_t = f(w_t) = [1, w_t, w_t^2, \dots, w_t^r]$

3] Chebyshev Polynomials: $w_t \in [-1,1]$
- $x_t = f(w_t) = [\cos(j\cos^{-1}(w_t))]_{j=0}^{r}$
  $= [1, w_t, 2w_t^2 - 1, 4w_t^3 - w_t, \dots]$
- basis elements are almost orthonormal

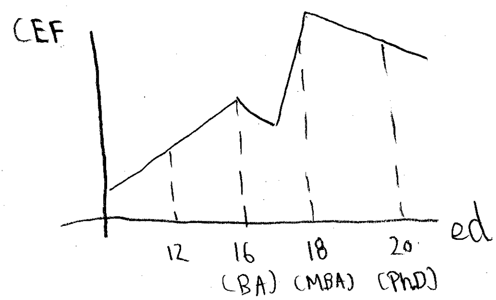
- RMSAE $= \left[E\left[(g(w_t) - f(w_t)'\beta)^2\right]\right]^{\frac{1}{2}}$
  root mean square approximation error
- MAE $= \max_w |g(w) - f(w)'\beta|$
  maximum approximation error

	Splines		Chebyshev	
	K=3	K=8	K=3	K=8
RMSAE	1.37	0.65	1.39	1.09
MAE	2.27	0.95	2.19	1.81

Example (1990 Census)

- $g(w) = E[\log(y)|w]$ = (CEF of log wages (y))

where $w \in \{8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20\}$



- Here, Chebyshev approximations do better than linear splines.

	Splines		Chebyshev	
	K=3	K=8	K=3	K=8
RMSAE	0.12	0.08	0.12	0.05
MAE	0.29	0.17	0.30	0.12

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as $K \to +\infty$, does RMSAE $\to 0$? Yes.

- if so, how fast? Depends on the complexity of the functions.

Theorem 1.1: If (i) $g(w)$ is s-times continuously differentiable with uniformly bounded derivatives (by a constant M), then using K terms in either spline or power series approximations $x = f(w)$ as defined above gives

$$\min_b \left[E[(g(w) - x'b)^2] \right]^{1/2} \leq \text{const}_M \cdot K^{-s/d}, \text{ where}$$

$d = \dim(w)$

Remark: $\left| \frac{\partial^s(w)}{(\partial w)^s} \right| \leq M \quad \forall w \in \text{supp}(w_t)$