E.g. of T/F/U questions

1] True- False- Uncertain:

$x = e + \varepsilon \qquad \sim N(0, \sigma^2)$

- with risk-neutral agent / principal, can always get FB - make agent residual claimant. (true)
- with risk-averse agent / RN principal, can get FB - (uncertain) - can use Mirrlees scheme to approximate arbitrarily the FB

2] Adverse selection + MLRP $\Rightarrow$ monotone quantity schedule

- False - need monotone hazard rate

Problem Set 5:

Question 1:

- B - buyer   1 unit
- S - seller
- q - quality
- $v = v(q) + \theta$, $\theta \sim U[0,1]$
- $c = c(q)$
  - c, v non-verifiable
  - θ observed ex post by both parties
    - after observing θ, S chooses q, which is observed by B (but is not verifiable.)

- can choose to contract before or after q is chosen.

1] Contract before - choose P, cannot renegotiate, but can veto (ie deliver 0)

2] Ex post bargaining - either B makes take-it-or-leave-it offer. Alternatively, S makes TIOLI offer

a) If parties choose ex post with B making TIOLI offer, what is q choice?

b) If S makes TIOLI offer?

c) If ex ante bargaining? If $v(q)=q$ and $c(q)=\frac{q^2}{2}$

d) $\theta \sim f(\theta)$ general distribution.

a) If B makes TIOLI offer, B will always choose $p=0 \Rightarrow$ seller's payoff will be $-c(q) \Rightarrow$ seller will choose $q=0$.
- expected welfare is $v(0)+E[\theta]-c(0)$

b) If S makes TIOLI offer, will ask for $v(q)+\theta$. S's payoff is then $v(q)+\theta-c(q)$
- get FB: $v'(q)=c'(q)$

Expected welfare is $v(q^{FB}) + E[\theta] - c(q^{FB})$.

c) Ex ante contracting

If price is p, trade occurs if $v(q)+\theta \geq p$.

Then $q = \begin{cases} p-\theta & \text{if } p \geq \theta \text{ and } p \geq \frac{1}{2}(p-\theta)^2 \\ 0 & \text{else} \end{cases}$

Case 1: $0 < p < 1$

seller chooses $q = p - \theta$ when $p > \theta$ and $q = 0$ if $p < \theta$.

d) Only used distribution for parts a) and b).

Question 3:

Liquidation: 3 periods: 0, 1, 2

- investment at $t=0$ - purchase assets at cost I
- 1^{st} cash flow at $t=1$ - cash flow $= \begin{cases} x & p \\ 0 & 1-p \end{cases}$
- 2^{nd} cash flow at $t=2$ - deterministic payoff $y = f(\underbrace{A}_{\text{assets in place}} - \underbrace{L}_{\text{liquidation from before}})$
- Can liquidate at $t=1$
- assume $A < I$.
- assume: $y=0$ if $A=L$ $\quad f(0)=0$

  $y>0$ if $A>L$ $\quad f(x)>0 \ \forall x \neq 0$
- assume $f' > 1$ everywhere.
  - $f(A-L) - L \Rightarrow$ FOC: $f'(A-L^*) = 1 \Rightarrow$ FOC never satisfied $\Rightarrow$ No interior solution. Want $L=0$.

- at $t=2$, run away with profits.
- contract that specifies $(r_x, L_x), (r_0, L_0)$
  - r_x - pay r_x and liquidate L_x when x occurs
  - r_0 - pay r_0 and liquidate L_0 when 0 occurs
  - want truthful revelation of state by firm.

a) Set up the program

b) Show that $r_x > 0,\ L_x = 0$
$r_0 = 0,\ L_0 > 0$

Firm wats to

$$\max_{(r_x, L_x), (r_0, L_0)} \quad p[x - r_x + f(A - L_x)] + (1-p)[-r_0 + f(A - L_0)]$$

$$\text{s.t.} \quad p(r_x + L_x) + (1-p)(r_0 + L_0) \geq I \quad \text{(Breakeven)}$$

$$\left.\begin{array}{l} r_x \leq x \\ r_0 \leq 0 \\ 0 \leq L_x, L_0 \leq A \end{array}\right. \quad \text{(Feasibility Constraints)}$$

$$x - r_x + f(A - L_x) \geq x - r_0 + f(A - L_0) \quad (I(x))$$

$$-r_0 + f(A - L_0) \geq -r_x + f(A - L_x) \quad (I(0))$$

$$(I(x)) + (I(0)) \Rightarrow \quad r_x - r_0 = f(A - L_x) - f(A - L_0)$$

FB is clearly $L_x = L_0 = 0$ and r_x, r_0 s.t.

$$p r_x + (1-p) r_0 = I,$$

SB: Want to set r_x, r_0 as big as possible and L_x, L_0 as small as possible.

$r_x > r_0 \Leftrightarrow L_x < L_0$ (1)

$r_x = r_0 \Leftrightarrow L_x = L_0$ (2)

$r_x < r_0 \Leftrightarrow L_x > L_0$ (3)

Need $r_0 = 0 \Rightarrow$ (3) is not feasible.

$\Rightarrow$ (2) would not be optimal

Thus, $r_x > 0$ and $L_x < L_0$ need to be set to satisfy IC and break-even constraint for investor.