

Moore - Repullo (EMA '88)

- Subgame Perfect Implementation

- Truthful revelation is the unique equilibrium of an appropriately defined mechanism.

Distinction between observability and verifiability doesn't make sense in light of the Moore - Repullo paper. (Maskin - Tirole Restud '99)

B (Buyer), S (Seller)

- $U_B = v - p$

- $U_S = p - v = p$

- normalize $v = 0$

- Suppose $v \in \{\underline{v}, \bar{v}\} = \{10, 14\}$

1] B makes announcement "High" or "Low"

- If high, B pays S 14, and we stop

- If low, S can challenge or not challenge

2] If S does not challenge, B pays 10

3] If S challenges,

a) B pays a fine $F = 9$ to third party T

b) B is offered the good for 6

c) If B accepts, then S gets $F = 9$ from T and they trade for 6

d) If B rejects then S pays $F = 9$ to T

e) Nash bargain over good and then stop
 (*) The whole time, they both know what v is.
 It turns out that this mechanism yields truthful revelation.

Is this result robust to adding an arbitrarily small amount of incomplete information?

Aghion - Judenberg - Holden (2006)

Prior: $\circ \Pr [v=14] = p$
 $\circ \Pr [v=10] = 1-p$

$\circ \theta \in \{\theta', \theta''\}$ independently drawn θ . θ' - corresponds to $v=14$
 θ'' - corresponds to $v=10$

	$\theta'_B \theta'_S$	$\theta'_B \theta''_S$	$\theta''_B \theta'_S$	$\theta''_B \theta''_S$
$v=14$	$p(1-\epsilon)^2$	$p(1-\epsilon)\epsilon$	$p\epsilon(1-\epsilon)$	$p\epsilon^2$
$v=10$	$(1-p)\epsilon^2$	$(1-p)(1-\epsilon)\epsilon$	$(1-p)\epsilon(1-\epsilon)$	$(1-p)(1-\epsilon)^2$

common p -belief (Monderer - Samet, 1999)

$$\begin{aligned}
 & \text{(ie: } \Pr [\theta_B = \theta'_B, \theta_S = \theta'_S, v=14] \\
 &= \Pr [\theta_B = \theta'_B, \theta_S = \theta'_S | v=14] \Pr [v=14] \\
 &= \Pr [\theta_B = \theta'_B | v=14] \Pr [\theta_S = \theta'_S | v=14] \Pr [v=14] \\
 &= (1-\epsilon)(1-\epsilon) \cdot p = p(1-\epsilon)^2 \quad)
 \end{aligned}$$

announce signal	H High	L Low
θ_B'	$1 - \sigma_B'$	σ_B'
θ_B''	σ_B''	$1 - \sigma_B''$

action signal	C Challenge	NC Not Challenge
θ_S'	$1 - \sigma_S'$	σ_S
θ_S''	σ_S''	$1 - \sigma_S''$

- B will condition information on signal
- S will condition information on signal and B's action

$$\Pr[\theta_S = \theta_S' \mid \theta_B = \theta_B'] = \frac{p(1-\epsilon)^2 + (1-p)\epsilon^2}{p(1-\epsilon)^2 + (1-p)\epsilon^2 + \epsilon(1-\epsilon)}$$

$$\Pr[\theta_S = \theta_S'' \mid \theta_B = \theta_B'] = \frac{\epsilon(1-\epsilon)}{p(1-\epsilon)^2 + (1-p)\epsilon^2 + \epsilon(1-\epsilon)}$$

$$\Pr[\theta_B = \theta_B' \mid \theta_S = \theta_S', L] = \frac{\sigma_B [p(1-\epsilon)^2 + (1-p)\epsilon^2]}{\sigma_B [p(1-\epsilon)^2 + (1-p)\epsilon^2] + (1-\sigma_B)\epsilon(1-\epsilon)}$$

$$v_B(L \mid \theta_B = \theta_B') = \Pr[\theta_S = \theta_S' \mid \theta_B = \theta_B'] \cdot$$

$$[(1-\sigma_S')(\Pr[v=14 \mid \theta_B', \theta_S'])(14-9-6) + \Pr[v=10 \mid \theta_B', \theta_S'])(10-9-5) + \sigma_S' (E[v \mid \theta_B', \theta_S'] - 10)]$$

$$+ \Pr[\theta_S = \theta_S'' \mid \theta_B = \theta_B'] \cdot$$

$$[\sigma_S'' (\Pr[v=14 \mid \theta_B', \theta_S''])(14-9-6) + \Pr[v=10 \mid \theta_B', \theta_S''])(10-9-5) + (1-\sigma_S'') (E[v \mid \theta_B', \theta_S''] - 10)]$$

Proposition 1: Suppose that $p < \frac{14}{19}$. Then there is no sequence of equilibrium strategies σ_B, σ_S such that $\sigma_B' \rightarrow 0, \sigma_B'' \rightarrow 0$ as $\epsilon \rightarrow 0$.