Dynamic Moral Hazard

1. Can we improve on the static optimal solution?
   - Can repeated interaction improve welfare?
2. Is there an advantage of long-term contracts over a series of short-term contracts?
3. Can we put more structure on the moral hazard problem? (eg HM 87 - linear contracts)

Classification

1. Repeated output: $T \to +\infty \Rightarrow$ law of large numbers

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} (e^* + \varepsilon_t) = E[e^* + \varepsilon_t] = e^* \quad \text{(assuming } E[\varepsilon_t]=0\text{)}$$

- Let $Y_T = \frac{1}{T} \sum_{t=1}^{T} y_t$
- Let $w_T = \begin{cases} w^* & Y_T \in [y^* - a, y^* + a] \\ -K & \text{otherwise} \end{cases}$, and we can have $a \to 0$ as $T \to +\infty$

2. Repeated consumption
   - 2 periods $\Rightarrow q_1$ in pd 1 & q_2 in pd 2
   - 2 outcomes in each period. (ie $q_t \in \{q_L, q_H\}$)
     - $P_r[q_t = q_H | a_t] = p(a_t)$, where $a \in A = [0, \bar{a}]$
   - suppose $u' > 0$, $u'' < 0$, P risk neutral
   - No access to borrowing or savings for agent
   - $\{\underbrace{w_L, w_H}_{\text{first period}}, \underbrace{w_{LL}, w_{HL}, w_{LH}, w_{HH}}_{\text{second period}}\}$ optimal contract.

Claim: $$\frac{1}{u'(w_i)} = \frac{p(a_2 | h_2)}{u'(w_{iH})} + \frac{1 - p(a_2 | h_2)}{u'(w_{iL})}$$

Proof: Consider $(\hat{w}_i, \hat{w}_{ij}) \equiv \hat{w}$

where $u(\hat{w}_i) = u(w_i) - \varepsilon_i$ and $u(\hat{w}_{ij}) = u(w_{ij}) + \varepsilon_i$ } same expected utility for the agent

$\varepsilon_i > 0$ small

∘ agent will accept the contract.

$\Rightarrow$ leaves action plan unaffected

Principal's payoff in changing from $\hat{w}$ to w

∘ $$\Delta\Pi^P = p(a_1)\left[(w_H - \hat{w}_H) + p(a_2|\text{success at }1)(w_{HH} - \hat{w}_{HH}) + (1-p(a_2|\text{success}))(w_{HL} - \hat{w}_{HL})\right] + (1-p(a_1))\left[(w_L - \hat{w}_L) + p(a_2|\text{failure at }1)(w_{LH} - \hat{w}_{LH}) + (1-p(a_2|\text{failure}))(w_{LL} - \hat{w}_{LL})\right]$$

We know that $w_i - \hat{w}_i \approx \dfrac{\varepsilon_i}{u'(w_i)}$

and $w_{ij} - \hat{w}_{ij} \approx -\dfrac{\varepsilon_i}{u'(w_{ij})}$

$$\Rightarrow \Delta\Pi^P \approx p(a_1)\Bigg[\underbrace{\frac{1}{u'(w_H)} - \frac{p(a_2|\text{success})}{u'(w_{HH})} - \frac{(1-p(a_2|\text{success}))}{u'(w_{HL})}}_{A}\Bigg]\varepsilon_H + (1-p(a_1))\Bigg[\underbrace{\frac{1}{u'(w_L)} - \frac{p(a_2|\text{failure})}{u'(w_{LH})} - \frac{(1-p(a_2|\text{failure}))}{u'(w_{LL})}}_{B}\Bigg]\varepsilon_L$$

∘ want to choose $\varepsilon_H, \varepsilon_L$ small s.t. $\varepsilon_H A > 0$, $\varepsilon_L B > 0$

(ie $\text{sgn}(\varepsilon_H) = \text{sgn}(A)$, $\text{sgn}(\varepsilon_L) = \text{sgn}(B)$)

The only way for this to be not a profitable deviation is for $A = B = 0$, which was the desired result.

Thus, $$\frac{1}{u'(w_i)} = \frac{p(a_2 | h_2)}{u'(w_{iH})} + \frac{(1-p(a_2 | h_2))}{u'(w_{iL})} \quad i = H, L$$

- this exhibits "memory"

1. Not terribly simple
2. Not sure about improvement over second best.
3. What is the form of the optimal contract?
   - Obviously, due to "memory," we have that the long-term contract is superior to a sequence of short-term contracts.

Because the agent does not have access to credit, the principal, in effect, acts as a bank.

- are long-term contracts still optimal if the agent has access to credit?

Malcomson and Spinnewyn: assume P can observe A's saving/borrowing

- wages controlled
- borrowing/savings can be controlled
  - This control has memory.

⇒ wage contracts need not have memory
⇒ can use series of short-term contracts

Holmstrom-Milgrom involves repeated output and repeated actions. (1987) Hellwig-Schmidt: (2002)

1 period model → 2 period model → T period model

⇒ linearity in accounts ≠ linearity in profits

= for 2 output levels

- Discrete time highlights this difference b/t linearity in accounts and linearity in profits.
- Consider N+1 output levels
- $T=1$: $x \in \{x_0, \ldots, x_N\}$
  - $p = \{p_0, \ldots, p_N\} \in \Delta\{x_0, \ldots, x_N\} \equiv P$
  - $c(p)$ - cost of prob. distribution
  - $s(x)$ sharing contract
  - $U^P = x - s(x)$

    $U^A = -\exp\{-r(s - c(p))\}$ $r > 0$ coefficient of absolute risk aversion
  - $$\max_{s,p} \sum_{i=0}^{N} (x_i - \underbrace{s_i}_{s(x_i)}) p_i$$

    $$\text{s.t.} \quad \sum_{i=0}^{N} -\exp\{-r(s_i - c(p))\} p_i \geq -\exp\{-r\underset{\text{outside wage}}{w}\} \quad \text{(IR)}$$

    $$\text{and } p \in \underset{\hat{p} \in P}{\text{argmax}} \sum_{i=0}^{N} -\exp\{-r(s_i - c(\hat{p}))\} \hat{p}_i \quad \text{(IC)}$$
- This has a solution if P compact, c continuous. If (s^*, p^*) solves for given w, then $(s^* + w' - w, p^*)$ for w'. w doesn't affect p^*