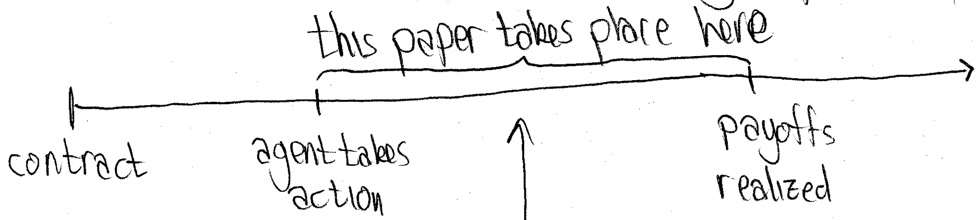
Dynamic Moral Hazard

Renegotiation, Career concerns.

Fudenberg, Tirole (EMA, 1990) - Renegotiation

- Two outputs $q_1 < q_2$ $(q_j \in \mathbb{R})$
- Two actions $a_L < a_H$
- $Pr[q = q_2 | a_H] \equiv p_H$
  $Pr[q = q_2 | a_L] \equiv p_L$
- cost of effort $\psi(a_H) \equiv k > 0 = \psi(a_L)$
- action not observable by principal



there may be renegotiation
- at this point, the action has already been taken - would be P.O. to give full insurance now
  - agent would anticipate this

- $t=0$: contract $w_1(\hat{a})$, $w_2(\hat{a})$
- $t=1$: agent chooses

$$a = \begin{cases} a_H & \text{w/pr } x \\ a_L & \text{w/pr } 1-x \end{cases}$$ nondegenerate randomization

- $t=2$: principal renegotiates and offers $\hat{w}_1(\hat{a})$, $\hat{w}_2(\hat{a})$, where $\hat{a}$ is announced effort.

- $t=3$: payoffs are realized, and payments are made

Suppose P wants to implement a_H

(*) Without loss of generality, we can restrict ourselves to renegotiation-proof contracts.

- usual screening logic: IC is binding for $a=a_L$

  $\Rightarrow w_1(a_L) = w_2(a_L) = w^*$

- also, if $x=0$, then $w_1(a_H) < w_2(a_H)$
- Furthermore, $u(w^*) = p_L u(w_2(a_H)) + (1-p_L) u(w_1(a_H))$

  since $p_L u(w_2(a_L)) + (1-p_L) u(w_1(a_L))$ (ICL)

  $\geq p_L u(w_2(a_H)) + (1-p_L) u(w_1(a_H))$

  - this will bind

- It turns out that $\exists$ a maximum value of x, say $x(w^*)$ that can be induced by a renegotiation proof (RP) contract.

- Ex ante IC: $p_H u(w_2(a_H)) + (1-p_H) u(w_1(a_H)) - K = u(w^*)$

Note: If $x \neq 0$ and $x \neq 1$, then agent is indifferent, because she anticipates no renegotiation, and principal, expecting agent to choose stipulated x, will not renegotiate.

IC + Ex ante IC jointly determine $w_2(a_H)$ and $w_1(a_H)$ as a function of w^*.

- Then P solves

$$\max_{w^*} \; x^* \; p_H(q_2 - w_2(w^*)) + (1-p_H)(q_1 - w_1(w^*)) + (1-x^*)\,[\,p_L q_2 + (1-p_L) q_1 - w^*\,]$$

s.t. (1) IR
(2) IC
(3) RP.

- Suppose P increases w^* by dw^* small.

<u>Claim</u>: P provides better insurance to type H, while still satisfying the ex post IC. This gives

$w_2 \to w_2 + dw_2$, $dw_2 < 0$
$w_1 \to w_1 + dw_1$, $dw_1 > 0$. The constraint is:

$$x(w^*)[p_H dw_2 + (1-p_H) dw_1] = (1 - x^*(w))\, dw^* \qquad (RP)$$

Final step: If we know $w_1(w^*)$ and $w_2(w^*)$, then we can find $x(w^*)$.

Hermalin-Katz (EMA '91)

- action is observed by P after it is taken, but before the revelation of uncertainty.

  ⇒ you can get the FB. (seems obvious)

Career concerns

Reprinted in Restud 99 (Holmstrom)

- Fama (JPE, 1980) - relational concerns can lead to FB
  - Handwavy

2 periods

Risk neutral employer (E) and a risk-neutral manager (M)

- $y_t = \theta + a_t + \varepsilon_t$
  - θ - ability, $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$
  - ε_t is white noise $\theta \sim N(0, \sigma_\varepsilon^2)$
  - a_t is action
  - $\theta, \varepsilon_1, \varepsilon_2$ are statistically independent
- There is a market for M's services in period 2.
  - get wage w_2
- cost of effort $\psi(a)$. assume $\psi(0)=0$, $\psi' > 0$, $\psi'' > 0$
- Discount factor δ
- market can observe y_1, y_2. Not verifyable.
  - cannot contract on output (eliminates issues of residual claimancy)
  - implies fixed wage in each period.

- clearly, $a_2 = 0$
- $w_2 = E[y_2 \mid \text{stuff}]$

$$= E[\theta \mid \text{stuff}]$$

$$= E[\theta \mid y_1 = \theta + a_1 + \varepsilon_1]$$

- assume that the market has rational expectations about a_1.
- Let a_1^* be the equilibrium value of a_1. Then

$$w_2 = E[\theta \mid \theta + a_1^* + \varepsilon = y_1]$$

$$= E[y_1 - a_1^* - \varepsilon]$$

$$= y_1 - a_1^*$$

(*) Optimal statistical decisions (De Groot) - Buy this book

- $$w_2 = \underbrace{\bar{\theta}}_{\text{prior}} \left(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\theta^2}\right) + (y_1 - a_1^*) \underbrace{\left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}\right)}_{\text{signal to noise ratio}}$$

$\Rightarrow$ M wants to

$$\max_{a_1} \left\{ w_1 + \delta E[w_2] - \psi(a_1) \right\}$$

$$= \max_{a_1} \left\{ w_1 + \delta \left[\bar{\theta}\left(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\theta^2}\right) + (\bar{\theta} + a_1 - a_1^*)\left(\frac{\sigma_\theta^2}{\sigma_\varepsilon^2 + \sigma_\theta^2}\right) \right] \right\}$$

$$= \max_{a_1} \left\{ \delta (a_1 - a_1^*) \left(\frac{\sigma_\theta^2}{\sigma_\varepsilon^2 + \sigma_\theta^2} \right) - \psi(a_1) \right\}$$

FOC: (a_1): $\delta \left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} \right) = \psi'(a_1)$

First best is given by $\psi'(a^{FB}) = 1$

$\Rightarrow \quad 0 < a_1^* < a_1^{FB}$.

since $\underbrace{\delta}_{<1} \underbrace{\left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} \right)}_{<1}$

Can show if $y_t = a_t + \theta + \varepsilon_t$, $t = \{1, 2, 3\}$

FOC is $\delta + \delta^2 = \psi'(a_1)$

- $a_1^* \uparrow$ if σ_θ^2 high or σ_ε^2 low
- Gibbons and Murphy - CEO incentive schemes.
  - career concerns become less powerful later in life.
- Can get different conclusions when you have multiple tasks.
  - can suggest what tasks should be clustered together.

- $E[\theta | y_1]$, $\theta \sim N$, $y_1 \sim N$

$$= E[\theta] + \frac{cov(\theta, y_1)}{Var(y_1)} [y_1 - E[y_1]]$$

$$= \bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2 + \sigma_\theta^2} [y_1 - \bar{\theta} - a_1^*]$$

since $Cov(\theta, y_1) = Cov(\theta, \theta + a + \varepsilon_1) = \sigma_\theta^2$

Tomorrow: relational contracts.