Moral hazard with multiple agents

Tournaments:

- use ordinal information/rankings to determine payoffs

Lazear-Rosen (JPE '81)

- 2 agents
  - $q_1 = a_1 + \varepsilon_1$, $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$
  - $q_2 = a_2 + \varepsilon_2$,
- Principal and agents are risk neutral
- contract: each agent gets t
  - winner gets prize w
    - i wins iff $q_i > q_j$
- agent cost of effort is $\psi_i(a_i) = \psi(a_i)$
- agent i solves

$$\max_{a_i} \{t + \Pr[\text{win}] \cdot w - \psi(a_i)\}$$

$$= \max_{a_i} \{t + \Pr[q_i > q_j \mid a_i] \cdot w - \psi(a_i)\}$$

$$= \max_{a_i} \{t + \Pr[a_i + \varepsilon_i > a_j + \varepsilon_j \mid a_i] \cdot w - \psi(a_i)\}$$

$$= \max_{a_i} \{t + \Pr[\varepsilon_j - \varepsilon_i < a_i - a_j \mid a_i] \cdot w - \psi(a_i)\}$$

$$= \max_{a_i} \{t + G(a_i - a_j) \cdot w - \psi(a_i)\}$$

where G is cdf of $\varepsilon_1 + \varepsilon_2 \sim N(0, 2\sigma^2)$

FOCs: (a_i): $g(a_i - a_j)w - \psi'(a_i) = 0$
(a_j): $g(a_j - a_i)w - \psi'(a_j) = 0$

Consider only symmetric equilibria: $a_i = a_j = a$
$\Rightarrow g(0)\cdot w = \psi'(a)$

What is first best? $\psi'(a^{FB}) = 1$
First best is attainable if $w = \frac{1}{g(0)}$

You can get the first-best with an ordinal measure.

Under risk aversion:
- tournaments good when common shocks big.
  - tournaments help filter out the noise when $y = a_i + \varepsilon_i + \eta$

Tournaments throw away the cardinal information.

Green-Stokey (JPE '83)
- Tournament with n agents
- Schedule of prizes $\{w_i\}_{i=1}^{n}$
- $\Pi = \sum_{i=1}^{n} (q_i - w_i)$
- $q_j = e_j + \varepsilon_j + \eta$
- $u(w_j) - c(e_j)$ $\quad u' \geq 0,\ u'' \leq 0$; $c' \geq 0$ $c'' \geq 0$

Focus on symmetric equilibrium: $e_i^* = e^* \ \forall i$

Then $EU = \frac{1}{n} \sum_{i=1}^{n} [u(w_i) - c(e^*)]$ since if $e_i^* = e^*$, then all prizes are equally likely.

Principal wants to:

$$\max_{e, \{w_i\}_{i=1}^{n}} E\left[\sum_{i=1}^{n} (q_i - w_i)\right]$$

$$\text{s.t. } \frac{1}{n} \sum_{i=1}^{n} u(w_i) - c(e^*) \geq \bar{u} \qquad \text{(IR)}$$

$$c'(e) = \sum_{i=1}^{n} \frac{\partial}{\partial e} \underbrace{\varphi_i(e, e^*)}_{\text{order statistic}} u(w_i)$$

where $\varphi_i(e, e^*) \equiv$ prob coming in in place i when i exerts e and everyone else exerts e^*.

Results:

1. Fix $n < +\infty$. Then, a relative performance evaluation (RPE) individual contract dominates the optimal tournament.
2. as $n \to +\infty$, the tournament's performance converges to the optimal RPE.

Tournaments help prevent the principal from cheating the agent.

(*) Conjecture: The reason why there is a nonmonotonic relationship b/t σ^2 and payoff is asymmetric case is suboptimality
- Information is always available under optimal contracts

Elimination Tournament: Rosen '86
- Moldovani-Sela (AER, 2001) asks, "what is the optimal prize structure in tournaments."
- Biggest increase in prizes should occur in the final period: incentive is based on $w_k - w_{k-1}$.

(*) Repeated tournaments.

Moral Hazard with multiple tasks

- One agent, one principle.

Grossman-Hart was agnostic about action set A.
- A compact.

Holmstrom-Milgrom (JLEO, '91) — Journal of Law, Economics, and Organization
- two tasks $i=1,2$
- $q_i = a_i + \varepsilon_i$, $\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\end{bmatrix} \sim N(0,\Sigma)$, $\Sigma = \begin{bmatrix}\sigma_1^2 & R\\ R & \sigma_2^2\end{bmatrix}$
- $u = -e^{-r(w-\psi(a_1,a_2))}$, $\psi(a_1,a_2) = \frac{1}{2}(c_1a_1^2 + c_2a_2^2) + \delta a_1 a_2$
  - $\delta > 0 \Rightarrow a_1, a_2$ are technological substitutes
  - $\delta < 0 \Rightarrow a_1, a_2$ are technological complements

- assume a linear contract is optimal:

$$w = t + v_1 q_1 + v_2 q_2$$

- Find CE: $\hat{w}(a_1,a_2) = E[w(a_1,a_2)] - \frac{r}{2} Var(w(a_1,a_2)) - \psi(a_1,a_2)$

$$= E[t + v_1(a_1+\varepsilon_1) + v_2(a_2+\varepsilon_2)]$$
$$- \frac{r}{2} Var(t + v_1(a_1+\varepsilon_1) + v_2(a_2+\varepsilon_2))$$
$$- \frac{1}{2}(c_1 a_1^2 + c_2 a_2^2) - \delta a_1 a_2$$

$$= t + v_1 a_1 + v_2 a_2 - \frac{r}{2}(v_1^2 \sigma_1^2 + v_2^2 \sigma_2^2 + 2 v_1 v_2 R)$$
$$- \frac{1}{2}(c_1 a_1^2 + c_2 a_2^2) - \delta a_1 a_2$$

assume $R = 0$.

agent: $\max_{a_1,a_2} \; t + v_1 a_1 + v_2 a_2 - \frac{r}{2}(v_1^2\sigma_1^2 + v_2^2 \sigma_2^2) - \frac{1}{2}(c_1 a_1^2 + c_2 a_2^2) - \delta a_1 a_2$

(a_1): $v_1 - c_1 a_1 - \delta a_2 = 0 \Rightarrow v_1 = c_1 a_1 + \delta a_2$

(a_2): $v_2 - c_2 a_2 - \delta a_1 = 0 \Rightarrow v_2 = c_2 a_2 + \delta a_1$

Principal: $\max_{v_1,v_2,a_1,a_2} \{ a_1 + a_2 - t - v_1 a_1 - v_2 a_2 \}$

s.t. $\hat{w}(a_1,a_2) = t + v_1 a_1 + v_2 a_2 - \frac{r}{2}(v_1^2 \sigma_1^2 + v_2^2 \sigma_2^2) \geq \bar{w}$ (IR)

$v_1 = c_1 a_1 + \delta a_2$ (IC)

$v_2 = c_2 a_2 + \delta a_1$

$$\max_{v_1,v_2,a_1,a_2} \left\{ a_1+a_2-\frac{r}{2}(v_1^2\sigma_1^2+v_2^2\sigma_2^2)-\frac{1}{2}(c_1a_1^2+c_2a_2^2)-\delta a_1a_2 \right\}$$

s.t. (IC)

FOCs: $1-r\sigma_1^2 v_1 c_1 - r\sigma_2^2 v_2\delta - v_1 = 0$

$$\Rightarrow v_1 = \frac{1-r\sigma_2^2 v_2\delta}{1+r\sigma_1^2 v_1 c_1}$$

$$v_2 = \frac{1-r\sigma_1^2 v_1\delta}{1+r\sigma_2^2 v_2 c_2}$$

$$\Rightarrow v_1 = \frac{1+r\sigma_2^2(c_2-\delta)}{1+r\sigma_1^2c_1+r\sigma_2^2c_2+r^2\sigma_1^2\sigma_2^2(c_1c_2-\delta^2)}$$

$$v_2 = \frac{1+r\sigma_1^2(c_1-\delta)}{1+r\sigma_2^2c_2+r\sigma_1^2c_1+r^2\sigma_1^2\sigma_2^2(c_1c_2-\delta^2)}$$

1] If $\delta=0$, we get the usual $v_1=\frac{1}{1+rc_1\sigma_1^2}$, $v_2=\frac{1}{1+rc_2\sigma_2^2}$

ie a_1, a_2 technologically independent.

2] Go from $\delta=1$ to $\delta=-1$ (ie tech substitutes to tech. complements) $\Rightarrow v_1\uparrow$, $v_2\uparrow$

3] as $\sigma_2^2\to+\infty$, $v_2\to 0$

$$v_1 \to \frac{r(c_2-\delta)}{rc_2+r^2\sigma_1^2(c_1c_2-\delta^2)}$$

4] What if $B(e_1,e_2) \neq e_1+e_2$ (or $B(q_1,q_2) \neq q_1+q_2$).

ie $B(e_1,e_2) = \min\{e_1,e_2\}$

What if there is intrinsic motivation?

3 things:

(a) $\sigma_2^2 \to +\infty$

(b) $B(\cdot)$ extremely complementary

(c) intrinsic motivation (ie $v_1 = v_2 = 0$ and $e_1^* = I_1$, $e_2^* = I_2$)

(d) $\delta = 1$

Then $v_1 \to 0$ and $v_2 \to 0$.

- principal gets $\min\{I_1, I_2\} > 0$
- can explain low-powered incentives.