$u(a, I) = G(a) + K(a) v(I)$

- $G(a_1) + K(a_1) v(I) \geq G(a_2) + K(a_2) v(I)$

$\Leftrightarrow G(a_1) + K(a_1) v(I') \geq G(a_2) + K(a_2) v(I') \quad \forall I, I'$.

Then preferences for perfectly certain actions are independent of income.

(i) Moving support $\Rightarrow$ can get first best.
- rule this out by saying that $\pi_i(a) > 0 \ \forall i \ \forall a \in A$.

(ii) "Unpleasant theorem" - Mirrlees
- If we can arbitrarily approximate the first best, why concern ourselves with the second best?
- This was based on certain distributions
- Limited liability / Legal limits can prevent this outcome
- Hyper-rationality

Moral hazard with multiple agents

Relative performance evaluation
- Holmstrom (Bell, 1982a)

assume two symmetric agents
- $q_1 = a_1 + \varepsilon_1 + \beta\varepsilon_2$ - output of agent 1
- $q_2 = a_2 + \varepsilon_2 + \beta\varepsilon_1$

- assume $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$

Risk neutral principal
Risk averse agents

- $u(a,w) = -e^{-r(w-\psi(a))}$, where $\psi(a) = \frac{1}{2}ca^2$

- $w_1 = t_1 + v_1 q_1 + u_1 q_2$
- $w_2 = t_2 + v_2 q_2 + u_2 q_1$
- Note that $u_1 = u_2 = 0$ is "no relative performance evaluation"

Principal solves:

$$\max_{a_1, t_1, v_1, u_1} \{E[q_1 - w_1]\}$$

$$\text{s.t. } a_1 \in \arg\max_a E[-e^{-r(w_1 - \frac{1}{2}ca^2)}] \qquad \text{(IC)}$$

$$E[-e^{-r(w_1 - \frac{1}{2}ca^2)}] \geq -e^{-r\bar{w}} \qquad \text{(IR)}$$

$$\widehat{w_1} - \frac{1}{2}ca_1^2 = t_1 + v_1 q_1 + u_1 q_2 - \frac{1}{2}ca_1^2$$
$$= t_1 + v_1(a_1 + \varepsilon_1 + \beta\varepsilon_2) + u_1(a_2 + \varepsilon_2 + \beta\varepsilon_1) - \frac{1}{2}ca_1^2$$

$$CE = t_1 + v_1 a_1 + u_1 a_2 - \frac{1}{2}ca_1^2 - \frac{r}{2}\sigma^2[(v_1 + \beta u_1)^2 + (u_1 + \beta v_1)^2]$$

where $Var(v_1(a_1 + \varepsilon_1 + \beta\varepsilon_2) + u_1(a_2 + \varepsilon_2 + \beta\varepsilon_1))$
$$= Var(v_1(\varepsilon_1 + \beta\varepsilon_2) + u_1(\varepsilon_2 + \beta\varepsilon_1))$$
$$= \sigma^2((v_1 + \beta u_1)^2 + (u_1 + \beta v_1)^2)$$

FOC: (maximize CE):

(a_1) $\quad v_1 - ca_1 = 0 \Rightarrow a_1 = \frac{v_1}{c}$

P solves

$$\max_{t_1, v_1, a_1} \left\{ \frac{v_1}{c} - \left(t_1 + v_1 \frac{v_1}{c} + u_1 a_2\right) \right\}$$

s.t. $CE = \bar{w}$

$$\Rightarrow \max_{v_1, u_1} \left\{ \frac{v_1}{c} - \frac{v_1^2}{c} - \bar{w} + \frac{v_1^2}{c} - u_1 a_2 + u_1 a_2 - \frac{r}{2}\sigma^2\left[(v_1 + \beta u_1)^2 + (u_1 + \beta v_1)^2\right] - \frac{1}{2} c \frac{v_1^2}{c^2} \right\}$$

$$\Leftrightarrow \max_{v_1, u_1} \left\{ \frac{v_1}{c} - \frac{v_1^2}{2c} - \frac{r}{2}\sigma^2\left[(v_1 + \beta u_1)^2 + (u_1 + \beta v_1)^2\right] \right\}$$

Trick: given $v_1 \Rightarrow u_1$ is determined to minimize risk. Then v_1 is set to trade-off risk and reward.

Take FOC first wrt u_1

(u_1): $2(v_1 + \beta u_1)\beta_1 + 2(u_1 + \beta v_1) = 0$

$$v_1 \beta + \beta^2 u_1 + u_1 + \beta v_1 = 0$$

$$u_1(1 + \beta^2) = -2\beta v_1$$

$$u_1 = -\frac{2\beta v_1}{1 + \beta^2}$$

Clearly, $\beta = 0 \Rightarrow u_1 = 0$.

Next, we can plug in u_1 and take FOC wrt v_1 and get:

$$v_1 = \frac{1}{1 + rc\sigma^2\left(\frac{(1-\beta^2)^2}{1+\beta^2}\right)}$$

Why do we not see relative performance evaluation in the real world?

(*) Bellcuck (HLS) suggests that it is theft that is the reason why CEOs are not subject to relative performance evaluation.

- Bertrand, Mullanaithan - "are CEOs rewarded for Luck?"

Moral Hazard in Teams

Holmstrom (Bell, 1982b) Bell ⇔ RAND

- n agents who choose actions $a_1, \dots, a_n$.
- These produce revenue $q(a_1, \dots, a_n)$ with q concave.
- agents' utility function is: $I_i - \psi_i(a_i)$ with $\psi_i(a_i)$ convex

First best: (spse can observe efforts):

$$\max_{a_1, \dots, a_n} \left\{ q(a_1, \dots, a_n) - \sum_{i=1}^{n} \psi_i(a_i) \right\}$$

FOC:

$$(a_i): \frac{\partial q}{\partial a_i} = \psi_i'(a_i) \quad \forall i.$$

Second best: assume a_i is observable only to agent i, but everyone sees q.

a <u>partnership</u> consists of sharing rules $s_i(q)$ s.t. $\sum_{i=1}^{n} s_i(q) = q$

Each agent solves

$$\max_{a_i} \{ s_i(q(a_i, a_{-i})) - \psi_i(a_i) \}$$

FOC:

$$(a_i): \quad s_i'(q(a_i, a_{-i})) \frac{\partial q(a_i, a_{-i})}{\partial a_i} = \psi_i'(a_i)$$

For first best, we need $s_i'(q) = 1 \ \forall i$

But we know that $\sum_{i=1}^{n} s_i'(q) = 1$.

- want everyone to be the residual claimant

Suppose we introduce an $(n+1)^{th}$ player we call the BB.

Let $s_i(q) = q(a^*) \quad \forall i = 1, \ldots, n$

$$s_{n+1}(q) = \sum_{i=1}^{n} F_i - n\underbrace{q(a^*)}_{\text{pay this to each agent.}}$$

Need: $\sum_{i=1}^{n} F_i + q(a^*) \geq nq(a^*)$

$$F_i \leq q(a^*) - \psi_i(a_i^*)$$

at First Best, $q(a^*) - \sum_{i=1}^{n} \psi_i(a_i^*) \geq 0$

Remarks:

1. BB wants project to fail
2. Collusion

Moral Hazard in teams is important in incomplete contracts

Random schemes

- Legros - Matthews (ReStat, 1993)
- Can get FB using a random schem

Suppose $n=2$, $a_i \in \{0,1,2\}$ $q = a_1 + a_2$, $\psi_i = \frac{a_i^2}{2}$

FB: $\max\ a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2}$

$(a_i): a_i^* = 1$

SB: agent 1 sets $a_1^* = 1$
agent 2 sets $a_2^* = 1$ w/ pr $1-2\delta$
$a_2^* = 2$ w/ pr δ
$a_2^* = 0$ w/ pr δ

Consider $S_2(q) = \frac{(q-1)^2}{2}$

$S_1(q) = q - \frac{(q-1)^2}{2}$

at a_1^*, payoff for 2 is $\Pi(a_2) = \frac{(1+a_2-1)^2}{2} - \frac{1}{2}a_2^2$

$\Pi_2(0) = \Pi_2(1) = \Pi_2(2) = 0$

Make player 1 pay player 2 a large fine if $q<1$ or $q>3$ $\Rightarrow a_1^* = 1$

as $\delta \to 0$, we can approximate the first best outcome

- We will have the same problems as in the budget breaker situation.