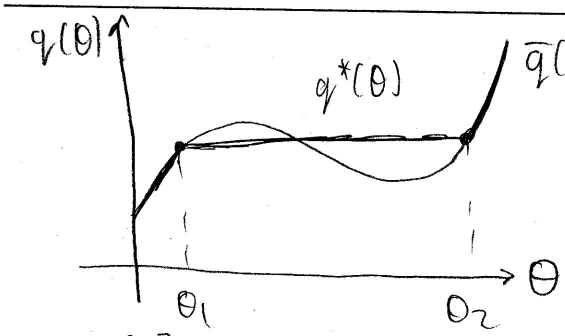



On regions where $\frac{\partial \bar{q}}{\partial \theta}(\theta) \geq 0$,

$$\bar{q}(\theta) = q^*(\theta)$$

Otherwise, $\frac{d\bar{q}}{d\theta}(\theta) = 0$

On $[\theta_1, \theta_2]$

$$\lambda(\theta) \leq 0, \quad \mu(\theta) = \frac{d\bar{q}(\theta)}{d\theta} = 0$$

$\lambda(\theta_1) = \lambda(\theta_2) = 0 \Rightarrow$ This is how you pin down $\theta_1$ & $\theta_2$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \left[\left[\theta - \frac{1}{h(\theta)}\right] v'(\bar{q}(\theta)) - c\right] d\theta = 0$$

- Two equations in two unknowns.

## Optimal Exclusion in Monopolistic Screening

- One monopolist: produces quality $s$ at cost $c(s) = s^2$
- (continuum of) consumers $\rightarrow$ unit demand:

$$u(\underbrace{\theta}_{\text{type}}|s) = \begin{cases} \theta s - T(s) & \text{if buys quality } s \\ 0 & \text{if not buy} \end{cases}$$

First best:

Seller: $\max\limits_{T_i, s_i} \{T_i - s_i^2\}$

$$\text{s.t. } \theta_i s_i - T_i \geq 0 \quad (IR)$$

$$\Rightarrow T_i = \theta_i s_i$$

$$\Rightarrow \max_{s_i} \{\theta_i s_i - s_i^2\} \Rightarrow (s_i): \theta_i - 2s_i = 0$$

$$\Rightarrow s_i = \frac{\theta_i}{2}$$

With two types: $\theta \in \{\theta_H, \theta_L\}$ $\Pr[\theta = \theta_H] = 1-\beta$

$\Pr[\theta = \theta_L] = \beta$

$$\max_{T_i, s_i} \beta[T_L - s_L^2] + (1-\beta)[T_H - s_H^2]$$

s.t.

$$\theta_H s_H - T_H \geq 0 \qquad IR_H$$
$$\theta_L s_L - T_L \geq 0 \qquad IR_L$$

$$\theta_H s_H - T_H \geq \theta_H s_L - T_L \qquad IC_H$$
$$\theta_L s_L - T_L \geq \theta_L s_H - T_H \qquad IC_L$$

- Only $IR_L$ / $IC_H$ will be binding:

$$\max \beta[\theta_L s_L - s_L^2] + (1-\beta)[\theta_H s_H - s_L(\theta_H - \theta_L) - s_H^2]$$

$$(s_L): \beta\theta_L - 2\beta s_L + (1-\beta)(\theta_L - \theta_H) = 0$$

$$\frac{\beta\theta_L - (1-\beta)(\theta_H - \theta_L)}{2\beta} = s_L$$

$$\Rightarrow \begin{cases} s_L = \max\left\{ \frac{\theta_L}{2} - \frac{1-\beta}{2\beta}(\theta_H - \theta_L), 0 \right\} \\ s_H = \frac{\theta_H}{2} \end{cases}$$

Trade-off between rents and efficiency

Assume $\theta \sim U[0,1]$

$$\max_{s(\theta),T(\theta)} \int_0^1 [T(\theta)-(s(\theta))^2]f(\theta)d\theta$$

$$\text{s.t. } \theta s(\theta) - T(\theta) \geq 0 \qquad \forall \theta$$

$$\theta s(\theta) - T(\theta) \geq \theta s(\theta') - T(\theta') \qquad \forall \theta, \theta'$$

Following the standard procedure,

$$\max_{s(\theta)} \int_0^1 \{[\theta s(\theta) - s(\theta)^2]f(\theta) - s(\theta)[1-F(\theta)]\} d\theta$$

$$(s(\theta)): [\theta - 2s(\theta)]f(\theta) - [1-F(\theta)] = 0$$

$$\Rightarrow s(\theta) = \frac{1}{2}\left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] = \frac{1}{2}(2\theta-1) \text{ since } \theta \sim U[0,1], \; F(\theta)=\theta$$

$$s(\theta) = \begin{cases} \theta - \frac{1}{2} & \theta \geq \frac{1}{2} \\ 0 & \theta < \frac{1}{2} \end{cases}$$

$\bar{\theta} = 1 \Rightarrow s(\bar{\theta}) = 1 - \frac{1}{2} = \frac{1}{2}$, which is optimal: $s(\bar{\theta}) = \frac{\bar{\theta}}{2}$

$\forall \theta < 1$, $\theta - \frac{1}{2} = \frac{2\theta - 1}{2} < \frac{\theta}{2} \iff 2\theta - 1 < \theta \iff \theta < 1$ ✓

Thus, $T(\theta) = \theta s(\theta) - \int_0^\theta s(x)dx \qquad s(\theta) = 0 \quad \theta < \frac{1}{2}$

$$\Rightarrow T(\theta) = \begin{cases} \theta s(\theta) - \int_{\frac{1}{2}}^{\theta} s(x)dx & \theta \geq \frac{1}{2} \\ 0 & \theta < \frac{1}{2} \end{cases}$$

$$= \begin{cases} \frac{\theta^2}{2} - \frac{1}{8} & \theta \geq \frac{1}{2} \\ 0 & \theta < \frac{1}{2} \end{cases}$$

Intel

486DX $\longrightarrow$ High quality

486SX $\longrightarrow$ Low quality (damaged)

Deneckere and McAfee (1996) – Damaged Goods

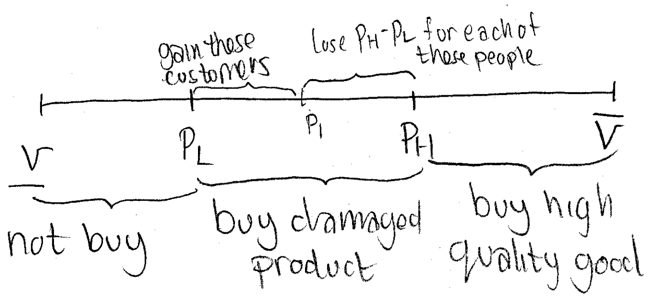
- First ten pages are filled with examples.

Monopolist, 2 qualities : $\underset{c_H}{v}$, $\underset{c_L}{\lambda(v)} < v$

where $c_L > c_H$.

- $c_H = p_1 - \dfrac{1-F(p_1)}{f(p_1)}$ for high quality mkt only
- $c_L = p_2 - \dfrac{1-F(\lambda^{-1}(p_2))}{f(\lambda^{-1}(p_2))}$ for low quality mkt only.

Could sell to both markets:




Is this set-up profitable?

When $\lambda(p_1) - c_L - \lambda'(p_1)\dfrac{1-F(p_1)}{f(p_1)} > 0$, it is profitable to sell the damaged good.

- sometimes, this can also be a Pareto improvement.

## Mirrlees Contracts

- Linear contracts combined with exponential utility and normal noise. This gives us:

  $w = \alpha + \beta q$ contract

  $q = a + \varepsilon$ $\quad \varepsilon \sim N(0, \sigma^2)$

  $$\Rightarrow \beta = \frac{1}{1 + \eta\sigma^2}$$

  - Even in this setting, linear contracts are not optimal.

Suppose $q = a + \varepsilon$, $\varepsilon \sim \underbrace{[-K, K]}_{\text{bounded support}}$. Here, we do not have a moral hazard problem due to the strange error structure. Let $a^*$ be the desired effort level. If $q \notin [a^* - \varepsilon, a^* + \varepsilon]$, punish agent severely. Otherwise, give agent $w^*$.

Assume $\varepsilon \sim N(0, \sigma^2)$. Then

$$\frac{f_a(q|a)}{f(q|a)} = \frac{q - a}{\sigma^2}$$

Spse $\exists$ $\underline{q}$ s.t. $q < \underline{q}$. Punish agent with $K$ (unbounded)

- offer $w(q) = w^* + \delta$, $\delta$ can be arbitrarily small
- IC: $\int_{-\infty}^{\underline{q}} u(K) f_a(q, a^*) dq + \int_{\underline{q}}^{+\infty} u(w^*) f_a(q, a^*) dq = \psi'(a^*)$
- Violates IR by $\int_{-\infty}^{\underline{q}} [u(w^*) - u(K)] f(q|a^*) dq$