Moral hazard

$A \subseteq \mathbb{R}$

$$\max_{a, I(\cdot)} \left\{ \int_{\underline{q}}^{\bar{q}} (q - I(q)) f(q|a)\, dq \right\}$$

s.t.

(i) $a \in \operatorname{argmax} \left\{ \int_{\underline{q}}^{\bar{q}} u(I(q)) f(q|a) dq - G(a) \right\}$ (IC)

(ii) $\int_{\underline{q}}^{\bar{q}} u(I(q)) f(q|a) dq - G(a) \geq \bar{u}$ (IR)

First-order approach: Replace (i) with

$$\int_{\underline{q}}^{\bar{q}} u(I(q)) f_a(q|a) dq = G'(a) \quad \text{(FOC)}$$

$$\int_{\underline{q}}^{\bar{q}} u(I(q)) f_{aa}(q|a) dq - G''(a) \quad \text{(SOC)}$$

Can form Lagrangian. The relevant K-T condition gives

$$\frac{1}{u'(I(q))} = \lambda + \mu \frac{f_a(q|a)}{f(q|a)}$$

for $I \uparrow$ in q, need $\dfrac{f_a(q|a)}{f(q|a)} \uparrow$ (MLRP)

Convexity of distribution function: $F_{aa} \geq 0$ along w/ MLRP

- allows us to replace (IC) with (FOC).
- F_{aa} holds for no commonly used distributions

agent is maximizing: $\int_{\underline{q}}^{\bar{q}} u(I(q))f(q|a)dq - G(a)$ (1)

- assume $I(\cdot)$ is differentiable

$$(1) \underset{\text{int. by parts}}{=} u(I(\bar{q})) - \int_{\underline{q}}^{\bar{q}} u'(I(q))I'(q)F(q|a)dq - G(a) \quad (2)$$

differentiate (2) twice wrt a:

$$-\underbrace{\int_{\underline{q}}^{\bar{q}} \underbrace{u'(I(q))}_{(+)}\underbrace{I'(q)}_{(+)\text{ by MLRP}}\underbrace{F_{aa}(q|a)}_{(+)\text{ by convexity}}dq}_{(-)} \underbrace{- G''(a)}_{(-)} < 0 \quad (3)$$

if MLRP and

Jewitt (EMA '88) provides conditions on u s.t. the FOCs are sufficient.

Grossman - Hart (EMA '83): different approaches
- eg "spanning condition"

Grossman - Hart

2 step approach:

1. Find lowest-cost way to implement a given action.
2. Choose the action which maximizes the difference between expected benefits and costs to the principal.

- $A = \{a_L, a_H\}$
- $q_1 < \dots < q_n$ outcome space
- I_i - payment to agent in state i
- $\pi : A \to \underbrace{S}_{\text{probability simplex}}$ $\quad (\pi_1(a_L), \dots, \pi_n(a_L)) = \pi(a_L)$
  $(\pi_1(a_H), \dots, \pi_n(a_H)) = \pi(a_H)$

- reservation utility $\bar{u}$
- $u = v(I) - G(a)$, $v' > 0$, $v'' < 0$, $\lim_{I \to \underline{I}} v(I) = -\infty$
  - Arrow: should have v bdd above and below

First best

$h \equiv v^{-1} \Rightarrow v(h(v)) = v$

$C_{FB}(a) = h(\bar{u} + G(a))$ cost of implementing a

since: $v(I) - g(a) = \bar{u} \Rightarrow v(I) = g(a) + \bar{u}$

$\Rightarrow I = h(\bar{u} + g(a))$

$$\max_{a \in A} \Big\{ \underbrace{\sum_{i=1}^n \pi_i(a) q_i}_{\text{expected benefit}} - \underbrace{C_{FB}(a)}_{\text{cost}} \Big\}$$

Second best: spse want to implement a_H

$$\min_{I_1, \dots, I_n} \Big\{ \sum_{i=1}^n \pi_i(a_H) I_i \Big\}$$

s.t. (IC) $\sum_{i=1}^n \pi_i(a_H) v(I_i) - G(a_H) \geq \sum_{i=1}^n \pi_i(a_L) v(I_i) - G(a_L)$

(IR) $\sum_{i=1}^n \pi_i(a_H) v(I_i) - G(a_H) \geq \bar{u}$

Define $w_i \equiv V(I_i)$

$$(*)\begin{cases} \Rightarrow \min_{w_1,\dots,w_n} \left\{ \sum_{i=1}^{n} \pi_i(a_H)\, h(w_i) \right\} \\ \text{s.t. (i) } \sum_{i=1}^{n} \pi_i(a_H) w_i - G(a_H) \geq \sum_{i=1}^{n} \pi_i(a_L) w_i - G(a_L) \\ \text{(ii) } \sum_{i=1}^{n} \pi_i(a_H) w_i - G(a_H) \geq \bar{u} \end{cases}$$

- This ensures we are doing a concave programming problem
- Does there exist a solution? The constraint set is unbounded.

Assume $\pi_i(a_H) > 0 \;\; \forall i$. Then $\exists$ a unique sol'n to $(*)$

- only reason there wouldn't be is if there was a sequence $\underbrace{\{(w_{1j}', \dots, w_{nj}')\}_{j=1}^{\infty}}$ $\Rightarrow \text{var}(I) \to +\infty$

  where $(w_{1j}', \dots, w_{nj}')$ solves the problem and some $w_{ij} \to +\infty$

  $\Rightarrow$ by risk aversion, we will violate IR.

<u>Claim 2:</u> $c(a_H) > c_{FB}(a_H)$ if $G(a_H) > G(a_L)$

- There is a welfare loss from the principal-agent problem.

<u>Claim 3:</u> (IR) is binding. Otherwise, we can lower all the w_i's w/o affecting IC.

FB=SB if

1] low action is optimal.

2] v is linear (risk neutrality for agent)

- make the agent the residual claimant
- if v concave, need to offer insurance to agent or you will violate IR.

3] If $\exists\ i$ s.t. $\pi_i(a_H)=0$ and $\pi_i(a_L)>0$

"moving support" - Mirrlees

- I will kill you if state i arises.

- In two action case, first-order approach is valid.
  - MLRP in $|A|=2$ case implies a monotonic incentive scheme.
  - MLRP if $|A|>2$ does not (alone) imply a monotonic incentive scheme.
- "internal inefficiency of firms": FB > SB.

2nd step

$$\max_{a\in A} \{ B(a) - \underbrace{c(a)}_{\text{need not be convex}} \} \quad (\text{eg. } B(a) = \sum_{i=1}^{n} \pi_i(a) q_i)$$

- This problem might not be concave since $c(a)$, the solution to 1st step, need not be well-behaved:

Can still do monotone (robust) comparative statics
- Milgrom-Shannon (EMA '94)

<u>Definition:</u> A set X is a <u>product set</u> if $\exists$ sets $X_1, \ldots, X_n$ s.t. $X = X_1 \times \ldots \times X_n$.

<u>Def^n</u> A function $f : X \to \mathbb{R}$ has <u>increasing difference</u> in (x_n, x_m) for $n \neq m$ iff $\forall\ x_n' \in X_n$ and $x_n'' \in X_n$ with $x_n' > x_n''$

$$f(x_1, \ldots, x_n', \ldots, x_N) - f(x_1, \ldots, x_n'', \ldots, x_N) \text{ is}$$

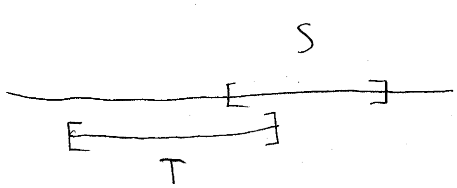
nondecreasing in x_m.

If f is differentiable, then f has increasing differences in x_n, x_m, $n \neq m$ if $\frac{\partial}{\partial x_n} f(\cdot)$ is nondecreasing in x_m. If $f \in C^2$, we require

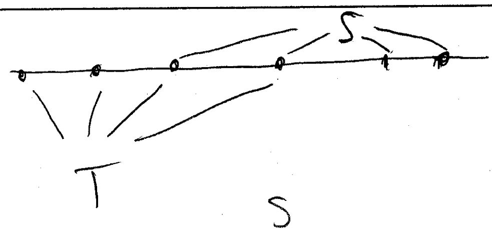
$$\frac{\partial^2 f}{\partial x_n \partial x_m} \geq 0.$$

<u>Defn</u> (Strong set order): A set $S \subseteq \mathbb{R}$ is said to be <u>as high</u> as $T \subseteq \mathbb{R}$ ($S \geq_s T$) iff

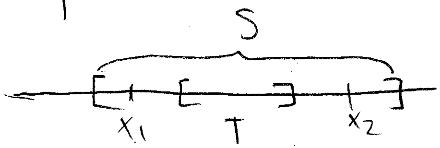
i) Each $x \in S - T$ is s.t. $x \geq y$ $\forall\ y \in T$ and
ii) Each $x' \in T - S$ is s.t. $x' \leq y$ $\forall\ y \in S$



$S \geq_s T$



$S \geq_s T$



$S \not\geq_s T$ since $x_1 \in S - T$ but $x_1 < x_2 \in T$

$T \not\geq_s S$ and $x_2 \in T - S$ but $x_2 > x_1 \in S$

- Risk neutral principal
- Risk averse agent
- Let $\varphi \in \mathbb{R}$ be a paramoter
- $q_1(\varphi), \ldots, q_n(\varphi)$
- $A = A_1 \times \ldots \times A_n$, $A_i \subseteq \mathbb{R}$, $A_i \neq \emptyset$ $\forall i$.
  - nonempty product set in $\mathbb{R}^n$
- $S = \{ y \in \mathbb{R}^n \mid y \geq 0, \sum_{i=1}^{n} y_i = 1 \}$
- Let $\pi \in C^2$ with $\pi : A \to S$

  $\pi(a) = (\pi_1(a), \ldots, \pi_n(a))$
- $u(a, I) = G(a) + K(a) V(I)$