Today: Last class on screening

Dynamic screening: Repeated interactions
- Types can be static
- Types can be dynamic. We will ignore this here.
  (B-D: §9.2)

$$\begin{array}{cc} \text{Durable} & \text{Non-durable} \\ \text{Commitment (C)} \quad NC & C \quad NC \end{array}$$

Durable goods:

- S (seller), B (buyer) 1 of each
- cost of production: $c = 0$
- B valuation $\bar{b}$ or $\underline{b}$ w/pr μ_1 and $1-\mu_1$
- assume $0 < \underline{b} < \bar{b}$
- assume: $\mu_1 \bar{b} > \underline{b}$
- common discount factor δ.

Case 1: Commitment (no renegotiation)
└ Revelation principle applies.
- If B announces $\bar{b}$, then with pr $\bar{x}_1$, B gets good today. With pr $\bar{x}_2$, B gets good tomorrow, pay price $= \bar{p}$.

- S solves

$$\max_{\bar{x}_1, \bar{x}_2, \underline{x}_1, \underline{x}_2} \left\{ \mu_1 \bar{p} + (1-\mu_1) \underline{p} \right\}$$

s.t. (i) $\bar{b}[\bar{x}_1(1+\delta) + (1-\bar{x}_1)\bar{x}_2\delta] - \bar{p} \geq \bar{b}[\underline{x}_1(1+\delta) + (1-\underline{x}_1)\underline{x}_2\delta]$.

(ii) $\underline{b}[\underline{x}_1(1+\delta) + (1-\underline{x}_1)\underline{x}_2\delta] - \underline{p} \geq 0$

Standard arguments will allow us to conclude that (i) and (ii) hold with equality.

Let $\overline{X}_1 \equiv \overline{x}_1 (1+\delta) + (1-\overline{x}_1)\overline{x}_2 \delta$

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Then $\underline{p} = \underline{b}\,\underline{X}_1$ from (ii)

$\overline{p} = \overline{b}\,\overline{X}_1 - \overline{b}\,\underline{X}_1 + \underline{b}\,\underline{X}_1$ from (i) and (ii)

$$\Rightarrow \max \left\{ \mu_1 \left[\overline{b}\,\overline{X}_1 - \overline{b}\,\underline{X}_1 + \underline{b}\,\underline{X}_1\right] + (1-\mu)\underline{b}\,\underline{X}_1 \right\}$$

s.t. $0 \le \overline{X}_1 \le 1+\delta$

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FOCs:

$-\mu_1 \overline{b} + (1-\mu_1)\underline{b} = 0$

$\underline{b} - \mu_1 \overline{b} < 0$

Conclusion: $\overline{X}_1 = 1+\delta$

$\underline{X}_1 = 0$

$\underline{p} = 0$

$\overline{p} = \overline{b} + \delta \overline{b}$

Charge high type all willing to pay and give them the commodity for sure.

Case 2: <u>No commitment</u>:

<u>Perfect Bayesian equilibrium</u>:

S could do one of three things:

$\boxed{1}$ Sell to both types at $t=1$

2] Sell to H at $t=1$, L at $t=2$

3] Never sell to low type

Under 1], $p = \underline{b} + \delta \underline{b}$

$\Rightarrow \Pi_{(1)} = \underline{b} + \delta \underline{b}$

Under 2], $p_2 = \underline{b}$, $p_1 = \bar{b} + \delta \underline{b}$ since

$\bar{b} + \delta \bar{b} - p_1 = \delta (\bar{b} - \underline{b})$ by IC

$$\Rightarrow \Pi_{(2)} = \mu_1 (\bar{b} + \delta \underline{b}) + (1-\mu_1) \delta \underline{b}$$
$$= \mu_1 \bar{b} + \mu_1 \delta \underline{b} + \delta \underline{b} - \delta \underline{b} \mu_1$$
$$= \mu_1 \bar{b} + \delta \underline{b}$$

$\Pi_{(2)} > \Pi_{(1)}$ if $\mu_1 \bar{b} > \underline{b}$

Under 3], sell to high type in both periods:

$p_1 = \bar{b} + \delta \bar{b}$, $p_2 = \bar{b}$

High type buys w/prob ρ_1 in pd 1
$1-\rho_1$ in pd 2

$$\Pr[\bar{b} \mid \text{declining first offer}] = \frac{\mu_1 (1-\rho_1)}{\mu_1(1-\rho_1) + (1-\mu_1)}$$

$$= \frac{\mu_1(1-\rho_1)}{1-\mu_1\rho_1} \equiv \sigma$$

S keeps price high if $\sigma \geq \underline{b}/\bar{b}$

$$\rho_1^* = \frac{\mu_1 \bar{b} - \underline{b}}{\mu_1(\bar{b} - \underline{b})}$$

$$\Rightarrow \Pi(3) = \mu_1 \rho_1 (\bar{b} + \delta \bar{b}) + \mu_1 (1-\rho_1) \delta \bar{b}$$
$$= \mu_1 \rho_1 \bar{b} + \mu_1 \delta \bar{b}$$
$$= \mu_1 \bar{b} \left[\frac{\mu_1 \bar{b} - \underline{b}}{\mu_1 (\bar{b} - \underline{b})} \right] + \mu_1 \delta \bar{b},$$

Recall: $\Pi(2) = \mu_1 \bar{b} + \delta \underline{b}$

$$\Rightarrow \Pi(3) > \Pi(2) \text{ iff } \mu_1 > \frac{\bar{b}\underline{b}(1+\delta) - \delta \underline{b}}{\delta \bar{b}^2 - \delta \bar{b}\underline{b} + \bar{b}\underline{b}} \equiv \bar{\mu}_2$$

Hart-Tirole (1988)

$\exists \; 0 \leq \bar{\mu}_1 \leq \bar{\mu}_2 \leq \ldots \leq \bar{\mu}_T < 1$ s.t.

$\mu_1 < \bar{\mu}_1$ sell to low types at $t=1$

$\bar{\mu}_2 > \mu_1 > \bar{\mu}_1$ sell to low types at $t=2$

$\vdots$

What happens as $\delta \to 1$? (ie time pds get shorter)

Coase: profits go to zero, Sell to low type immediately

Hart-Tirole (1988) $\bar{\mu}_i \to 1$ as $T \to \infty$

Non-durable goods

Remark 1: Under commitment, the solution is essentially the same as in the durable goods case.

Remark 2: Without commitment, the solution is very different, (if $T > 2$)

Ratcheting effect: don't want to reveal type at $t=1$ or else you will be charged high amt.

Remark 3: Screening fails because the price S has to charge in pd 1 to induce high types to buy is below the price at which the low types are prepared to buy.
(stems from a failure of the revelation principle. information "sepps out slowly.")

"Soft-budget constraint": Kornai (70's), Dewatripont-Maskin

Government G
Firms need 1 unit of capital
- α good or "quick" types: $R_g > 1$ (money)
  F_g (private benefit to manager)
- $1-\alpha$ bad or "slow" types: 0 money
  0 or negative private benefit

- Bad types can be refinanced at cost 1 and yield Π_b^* money
  E_b private benefit

  with $1 < \Pi_b^* + E_b < 2$

If govt commits, good type will volunteer, and we will get first best.

If not commit, everyone wants to volunteer in first rd. In second round, we have that gov't will refinance bad type

- Commitment is very important in those models.

Moral hazard

Principal hires agent to perform task. Can observe output but not action. Agent is risk averse.

- Owner/manager
- Client/doctor
- Citizens/government

Started by Arrow in 1960's.
Mark Pauly ('66)
Spence-Zeckhauser
Ross ('72?)
Mirrlees ('74, '75, '76)
Holmstrom ('79) - value of information (sufficient statistic)
Grossman-Hart ('83)

Basic setup: Principal-risk neutral, agent-risk averse.

agent: $a \in A$ = action set

- action leads to verifiable output q in some stochastic way. Think of $q \in \mathbb{R}$.
- $F(q;a)$

- incentive scheme $I(q)$

Principal solves:

$$\max_{\hat{I}(\cdot),\hat{a}} \left\{ \int [q - \hat{I}(q)] dF(q;\hat{a}) \right\}$$

s.t. $\hat{a}$ solves $\max_a \left\{ \int u(a, \hat{I}(q)) dF(q;a) \right\}$ (IC)

$$\int u(\hat{a}, \hat{I}(q))\, dF(q;\hat{a}) \geq \bar{u} \quad \text{(IR)}$$

- control problem for the principal

First-order approach:

$A \subseteq \mathbb{R}$

$u(a, I(\cdot)) = v(I) - c(a)$

Form a Lagrangian:

$$\frac{\partial \mathcal{L}}{\partial I} = 0 \Rightarrow \frac{1}{v'(I(q))} = \lambda + \mu \overbrace{\frac{f_a(q;a)}{f(q;a)}}^{\text{Likelihood ratio}}$$

$\Rightarrow I(q) \uparrow$ iff $\frac{f_a}{f} \uparrow q$ (monotone likelihood ratio property)

i.e. monotone incentive scheme if MLRP holds.