Indivisible public project

- value S for consumers
- cost : $c = \underbrace{\beta}_{\text{efficiency parameter}} - \underbrace{e}_{\text{effort}}$
- Find cheapest contract which allows project to be implemented
- utility of firm: $\overbrace{U}^{\text{profits}} = \underbrace{t}_{\substack{\text{transfer}\\\text{from gov't}}} - \underbrace{\psi(e)}_{\substack{\text{cost of}\\\text{effort}}}$ $\quad \psi' > 0$, $\psi'' > 0$
- λ shadow cost of public funds
- consumers' payoff is: $S - (1+\lambda)(t + \underbrace{\beta - e}_{\text{cost}})$
- utilitarian regulator: $W = S - (1+\lambda)(t+\beta-e) + t - \psi(e)$

$$= S - (1+\lambda)(\beta - e + \psi(e)) - \lambda U$$

Complete information:

- regulator: $\max\limits_{e} \; S - (1+\lambda)(\beta - e + \psi(e)) - \lambda U$

$$\text{s.t. } U \geq 0$$

(e): $\underbrace{\psi'(e^*) = 1}_{\text{efficient effort}}$ and $\underbrace{t^* = \psi(e^*)}_{\text{firm breaks even}}$ (no rents)

asymmetric information: assume $\beta \sim F(\beta)$ on $[\underline{\beta}, \overline{\beta}]$

- regulator: $\max\limits_{e(\beta), U(\beta)} \int_{\underline{\beta}}^{\overline{\beta}} [S - (1+\lambda)(\beta - e(\beta) + \psi(e(\beta))) - \lambda U] \underbrace{dF(\beta)}_{f(\beta)d\beta}$

$$\text{s.t. } u(\bar{\beta}) \geq 0 \qquad \text{(IR)}$$

$$u'(\beta) = -\psi'(e(\beta)) \qquad \text{(IC)}$$

$$e'(\beta) \leq 1 \qquad \text{(Monotonicity)}$$

Rewriting the objective function:

$$\max_{e(\beta)} \int_{\underline{\beta}}^{\bar{\beta}} \left[S - (1+\lambda)(\beta - e(\beta) + \psi(e(\beta))) - \lambda \frac{F(\beta)}{f(\beta)} \psi'(e(\beta)) \right] dF(\beta)$$

$$(e(\beta)): \; \psi'(e(\beta)) = 1 - \frac{\lambda}{1+\lambda} \frac{F(\beta)}{f(\beta)} \psi''(e(\beta))$$

Clearly, $\psi'(e(\underline{\beta})) = 1 \Rightarrow$ no distortions at the top since $F(\underline{\beta}) = 0$

Bunching and Ironing:

$$\max_{q(\theta), T(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} [T(\theta) - cq(\theta)] \, dF(\theta)$$

$$\text{s.t.} \quad \underline{\theta} v(q(\underline{\theta})) - T(\underline{\theta}) \geq 0 \qquad \text{(IR)}$$

$$T'(\theta) = \theta v'(q(\theta)) \quad \forall \theta \qquad \text{(IC)}$$

$$\frac{dq(\theta)}{d\theta} \geq 0 \quad \forall \theta \qquad \text{monotonicity}$$

Typically, we assume $\frac{dq(\theta)}{d\theta} \geq 0$ and verify that the solution satisfies this.

$$\left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] v'(q(\theta)) = c \iff g(\theta)\, v'(q(\theta)) = c \quad (*)$$

Does this satisfy $\frac{dq(\theta)}{d\theta}$?

- Sufficient condition: $h(\theta) = \frac{f(\theta)}{1-F(\theta)} \uparrow$ and $v'' < 0$

$$(*) \Rightarrow \frac{dq}{d\theta} = -\frac{g'(\theta)\,\overbrace{v'(q(\theta))}^{>0}}{\underbrace{v''(q(\theta))}_{<0}\,\underbrace{g(\theta)}_{>0}}$$

>0 since $c > 0$ and $v'(q(\theta)) > 0$.

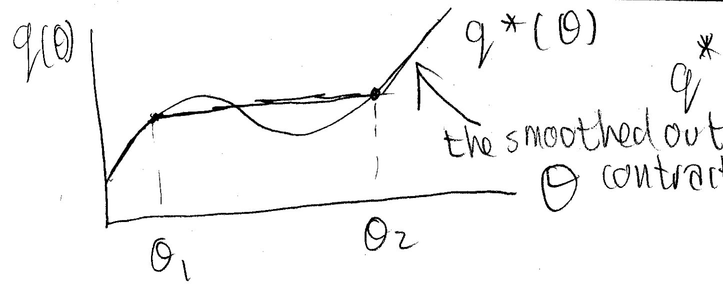
>0 iff $g'(\theta) > 0$

$$h'(\theta) = \frac{f'(\theta)(1-F(\theta)) + (f(\theta))^2}{(1-F(\theta))^2}$$

$$g'(\theta) = \frac{f'(\theta)(1-F(\theta)) + 2(f(\theta))^2}{(f(\theta))^2}$$

$$h'(\theta) > 0 \Rightarrow f'(\theta)(1-F(\theta)) + (f(\theta))^2 > 0$$
$$\Rightarrow f'(\theta)(1-F(\theta)) + 2(f(\theta))^2 > 0$$
$$\Rightarrow g'(\theta) > 0 \Rightarrow \frac{dq}{d\theta} > 0$$

Noldeke and Samuelson: optimal bunching without optimal control



the smoothed out contract is $\bar{q}(\theta)$

q^* given by $\left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] v'(q(\theta)) = c$

For some range, $\frac{dq^*(\theta)}{d\theta} < 0$. Need to impose $\frac{dq(\theta)}{d\theta} \geq 0$

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta v(q(\theta)) - cq(\theta) - \frac{v(q(\theta))}{h(\theta)}\right] dF(\theta)$$

s.t. $\frac{dq(\theta)}{d\theta} \geq 0$

assume
- objective function strictly concave in θ.
- $\frac{dq^*(\theta)}{d\theta}$ changes signs only a finite # of times.

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta v(q(\theta)) - cq(\theta) - \frac{v(q(\theta))}{h(\theta)}\right] dF(\theta)$$

s.t. $\frac{dq(\theta)}{d\theta} = \mu(\theta)$, $\mu(\theta) \geq 0$

The associated Hamiltonian is:

$$H(\theta, q, \mu, \lambda) = \left[\theta v(q(\theta)) - cq(\theta) - \frac{v(q(\theta))}{h(\theta)}\right] f(\theta) + \lambda(\theta)\mu(\theta)$$

1] $H(\theta, \bar{q}(\theta), \bar{\mu}(\theta), \lambda(\theta)) \geq H(\theta, \bar{q}(\theta), \mu(\theta), \lambda(\theta))$

2] $\frac{d\lambda(\theta)}{d\theta} = -\left[\left(\theta - \frac{1}{h(\theta)}\right) v'(\bar{q}(\theta)) - c\right] f(\theta)$

$\boxed{3}$ $\lambda(\underline{\theta}) = \lambda(\bar{\theta}) = 0$ (Transversality cond.)

$\boxed{4}$ $\mu(\theta) \geq 0$

Integrating (2):

$$\lambda(\hat{\theta}) = -\int_{\underline{\theta}}^{\hat{\theta}} [(\theta - \frac{1}{h(\theta)})v'(\bar{q}(\theta)) - c]f(\theta)d\theta$$

By transversality,

$$0 = \lambda(\bar{\theta}) = -\int_{\underline{\theta}}^{\bar{\theta}} [(\theta - \frac{1}{h(\theta)})v'(\bar{q}(\theta)) - c]f(\theta)d\theta$$

When $\lambda(\hat{\theta}) \leq 0$, $\bar{\mu}(\theta) = 0$

But $\bar{\mu}(\theta) = \frac{dq(\theta)}{d\theta}$

Thus, $\frac{d\bar{q}(\hat{\theta})}{d\theta} \int_{\underline{\theta}}^{\hat{\theta}} [(\theta - \frac{1}{h(\theta)})v'(\bar{q}(\theta)) - c]f(\theta)d\theta = 0$

If $\frac{d\bar{q}(\hat{\theta})}{d\theta} > 0$, $[\theta - \frac{1}{h(\theta)}]v'(\bar{q}(\theta)) = c$

$\Rightarrow \bar{q}(\theta) = q^*(\theta)$ on the intervals on which $\frac{d\bar{q}(\theta)}{d\theta} > 0$