Today: Finish off static screening
Next time: Dynamic screening.

Recall: (i) $\theta_i v(q_i) - T_i \geq \theta_i v(q_{i-1}) - T_{i-1} \quad \forall i = 2, \dots, n$

(ii) $q_i > q_{i-1}$

(iii) $\theta v(q_1) - T_1 \geq 0$

Now, suppose $\theta \in [\underline{\theta}, \overline{\theta}]$ with $F(\theta)$ as the cdf and $f(\theta)$ as the pdf

- Will offer $T(\theta), q(\theta)$ menu of contracts.

$$\max_{T(\cdot), q(\cdot)} \left\{ \int_{\underline{\theta}}^{\overline{\theta}} [T(\theta) - cq(\theta)] f(\theta) d\theta \right\}$$

s.t. (i) $\theta v(q(\theta)) - T(\theta) \geq \theta v(q(\hat{\theta})) - T(\hat{\theta}) \quad \forall \theta, \hat{\theta}$.

(ii) $\theta v(q(\theta)) - T(\theta) \geq 0 \quad \forall \theta$.

$T(\cdot), q(\cdot)$ is implementable if it satisfies (i).

Claim: an allocation $T(\theta), q(\theta)$ is implementable if and only if (i) $\theta v'(q(\theta)) \frac{dq(\theta)}{d\theta} - T'(\theta) = 0$

and (ii) $\frac{dq(\theta)}{d\theta} \geq 0$

(i) is referred to as the local adjacency condition
(ii) is the monotonicity condition.

Spence-Mirrlees condition (single-crossing property):
Let $u(q,\theta,T)$ be the agents' utility function. Then

$$\frac{\partial}{\partial\theta}\left[\frac{-\partial u/\partial q}{\partial u/\partial T}\right]>0$$

Here, we have $u(q,\theta,T)=\theta v(q)-T$.

$$\frac{-\partial u/\partial q}{\partial u/\partial T}=\theta v'(q) \Rightarrow \frac{\partial}{\partial\theta}\theta v'(q)=v'(q)>0$$

Rewriting the problem in light of the claim:

$$\max_{T(\cdot),q(\cdot)}\left\{\int_{\underline{\theta}}^{\overline{\theta}}[T(\theta)-cq(\theta)]f(\theta)d\theta\right\}$$

s.t. $T(\cdot),q(\cdot)$ is implementable
(i) $\theta v'(q(\theta))\frac{dq}{d\theta}-T'(\theta)\geq 0 \quad \forall\theta$
(ii) $\frac{dq(\theta)}{d\theta}\geq 0 \quad \forall\theta$
(iii) $\underline{\theta}\, v(q(\underline{\theta}))-T(\underline{\theta})=0$

Let $w(\theta)\equiv\theta v(q(\theta))-T(\theta)$

$$\frac{dw(\theta)}{d\theta}=\frac{\partial w(\theta)}{\partial\theta}=v(q(\theta)) \quad \text{by envelope thm}$$

$$\Rightarrow \int_{\underline{\theta}}^{\theta}v(q(t))dt+\underbrace{w(\underline{\theta})}_{=0 \text{ by (iii)}}=w(\theta)$$

Thus, the problem becomes:

$$\max_{q(\cdot)} \underbrace{\int_{\underline{\theta}}^{\bar{\theta}} \left[\theta v(q(\theta)) - \int_{\underline{\theta}}^{\theta} v(q(t))dt - cq(\theta)\right] f(\theta)d\theta}_{(1)}$$

$$\text{s.t.} \quad \frac{dq(\theta)}{d\theta} \geq 0 \quad \forall \theta$$

Recall: $\int_{\underline{\theta}}^{\bar{\theta}} uv' = [uv]_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} v'u$

Here, $v' = f(\theta)$

$u = \int v(q(t))dt$

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\underline{\theta}}^{\theta} v(q(t))dt\, F(\theta)\right|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta))F(\theta)d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} v(q(t))dt - \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta))F(\theta)d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} v(q(\theta))[1-F(\theta)]d\theta$$

$\Rightarrow$ (1) becomes:

$$\max_{q(\cdot)} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} (\theta v(q(\theta)) - cq(\theta))f(\theta) - v(q(\theta))[1-F(\theta)]d\theta \right.$$

<u>FOC:</u> $\theta v'(q(\theta)) = v'(q(\theta))\left(\frac{1-F(\theta)}{f(\theta)}\right) + c \quad \forall \theta$

(i) If $\theta = \bar{\theta}$, then $\bar{\theta}\, v'(q(\bar{\theta})) = c$

(ii) If $\theta < \bar{\theta}$, then $\theta\, v'(q(\theta)) > c \Rightarrow q$ is too low

For monotonicity to be satisfied, we need that

$\frac{f(\theta)}{1-F(\theta)} \uparrow \theta$

(*) In general, this is a sufficient condition.

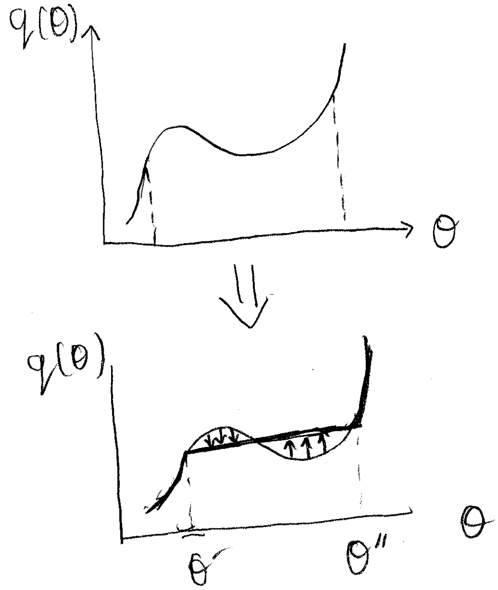
Monotone hazard rate property

(*) In this problem, this is a necessary and sufficient condition

- This holds for all log concave distributions:
  - Normal, uniform, exponential, F, T, Poisson, log-normal are all log concave.
  - We will return to this when we do moral hazard.

What if monotonicity condition failed?

- Ironing and bunching
- use optimal control theory.



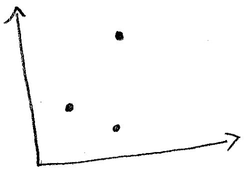
Random schemes

Maskin and Riley (RAND, '84)

- Two types $\theta_2 > \theta_1$
- Suppose people are risk averse and θ_2 is more risk averse
- Offer people a lottery of (T, q)

- Can potentially reduce informational rents to θ_2 type.

Multi-dimensional types:



cannot reduce an n dimensional type space to an n-1 dimensional typespace in an order preserving way, in general.

Laffont-Maskin-Rochet ('87)
- Bunching is more likely in a two dimension case

armstrong (EMA, '96) - n-dimensional type space
- Some agents are always excluded.

Rochet-Chone ('97)
- upward IC constraints can bind at the optimum
- random contracts can be optimal
- no generalization of monotone hazard rate property which rules out bunching.
  - There is a real loss in generality when we assume a one-dimensional type space.

armstrong ('97)
- If large number of independently valued dimensions, then optimal contract is approximated by a two-part tariff.

Mirrlees ('76 J Pub. E)

- If $\underbrace{m}_{\text{dim of types}} < \underbrace{n}_{\#\text{ commodities}}$, then can characterize the soln by a single elliptical equation which can be solved.
- If $m \geq n$, problem with system of partial differential equations.

Next time: Dynamic adverse selection.

Recitation:

- multidimensional screening