Screening

- 2 players
  - principal, agent
  - payoffs: A: $G(u(q,\theta)-T)$
    P: $H(v(q,\theta)+T)$

q is output (verifiable - enforceable in court)

θ is agents' private information

- Let $v(q,\theta) = -cq$
  $u(q,\theta) = \theta V(q)$

$\Rightarrow G(u(q,\theta)-T) = \theta V(q) - T, \quad V''<0, \; V'>0$

$H(v(q,\theta)+T) = T - cq$

- $\theta \in \Theta = \{\theta_1, \dots, \theta_n\}$ with $p_1, \dots, p_n$
- principal has all the bargaining power

Case 1: Ex ante, nobody knows θ,
Ex post, θ verifiable

P: $\max_{(q_i, T_i)} \sum_{i=1}^{n} p_i (T_i - cq_i)$

s.t. $\sum_{i=1}^{n} p_i(\theta_i V(q_i) - T_i) \geq \bar{U}$

Case 2: Both know θ. $\bar{U} \equiv 0$

$\max_{(q_i, T_i)} \{T_i - cq_i\}$

s.t. $\theta_i V(q_i) - T_i \geq 0$

$\Leftrightarrow \max_{q_i} \{\theta_i V(q_i) - cq_i\}$

FOC: (q_i): $\theta_i V'(q_i) = c$

cf 1^{st} degree price discrimination.

Second best:

A knows θ_i, B does not.

Suppose $\theta = \theta_2$

If reveal, get: $\theta_2 V(q_2^*) - T_2^* = 0$

If say θ_1, get: $\theta_2 V(q_1^*) - T_1^* = \theta_2 V(q_1^*) - \theta_1 V(q_1^*)$
$= (\theta_2 - \theta_1) V(q_1^*) > 0$

$\Rightarrow$ will not reveal oneself.

In general, second best with n types:

General problem.

$$\max_{T(q)} \left\{\sum_{i=1}^{n} p_i (T_i(q_i) - cq_i)\right\}$$

s.t. $\theta_i V(q_i) - T(q_i) \geq 0 \ \forall i$ (IR)

$q_i = \arg\max_q \{\theta_i V(q) - T(q)\} \ \forall i$ (IC)

This problem is a mess, but we can use the revelation principle:

$$\max_{(T_i, q_i)} \left\{\sum_{i=1}^{n} p_i (T_i - cq_i)\right\}$$

s.t. $\theta_i V(q_i) - T_i \geq 0 \quad \forall i$

$\theta_i V(q_i) - T_i \geq \theta_i V(q_j) - T_j \quad \forall j$

Let $n=2$: $\theta_H > \theta_L$

$$\max \{ p_H (T_H - cq_H) + p_L (T_L - cq_L) \}$$

s.t. (i) $\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L$ (IC_H)

(ii) $\theta_L v(q_L) - T_L \geq 0$ (IR_L)

- We will verify that we need not check IC_L and IR_H.
- Both constraints will bind:
  - IR_L will be binding, or P increases T_L
  - IC_H will be binding, or P increases T_H

$$\Rightarrow \max_{q_L, q_H} \{ p_H (\theta_H v(q_H) - \theta_H v(q_L)) + \theta_L v(q_L) - cq_H) + p_L (\theta_L v(q_L) - cq_L) \}$$

(q_H): $p_H \theta_H v'(q_H) - p_H c = 0$

$\Rightarrow \theta_H v'(q_H) = c \Rightarrow$ first best for θ_H type

(q_L): $-p_H \theta_H v'(q_L) + p_H \theta_L v'(q_L) + p_L \theta_L v'(q_L) - c p_L = 0$

$$\Rightarrow \theta_L v'(q_L) = \frac{c}{1 - \left(\frac{1-p_L}{p_L}\right)\left(\frac{\theta_H - \theta_L}{\theta_L}\right)} > c$$

$\Rightarrow q_L$ is less than the first best.

$\Rightarrow$ distort low-type bundle to get high type to reveal bundle.

High type gets more than reservation utility. This is an informational rent.

Mirrlees (1971)

- Optimal income tax
- $q = \mu e$ μ - ability, e - effort, q - output μ is private information

$\mu_L < \mu_H$

$\pi \quad 1-\pi$

- $u(q - T - \underbrace{\psi(e)}_{\text{cost of effort}})$: $\psi' > 0$ $\psi'' > 0$
- BC: $\pi T_L + (1-\pi) T_H \geq 0$

First best:

$$\max_{\substack{e_L, e_H \\ T_L, T_H}} \{\pi u(\mu_L e_L - T_L - \psi(e_L)) + (1-\pi) u(\mu_H e_H - T_H - \psi(e_H))\}$$

$$\text{s.t. } \underbrace{\pi T_L + (1-\pi) T_H \geq 0}_{\text{This will bind.}} \Rightarrow T_H = -\frac{\pi}{1-\pi} T_L$$

$$\Rightarrow \max_{e_L, e_H, T_L} \left\{\pi u(\mu_L e_L - T_L - \psi(e_L)) + (1-\pi) u\left(\mu_H e_H + \frac{\pi}{1-\pi} T_L - \psi(e_H)\right)\right\}$$

$$(T_L): -\pi u'(\mu_L e_L - T_L - \psi(e_L)) = u'\left(\mu_H e_H + \frac{\pi}{1-\pi} T_L - \psi(e_H)\right)$$

$$(e_L): \mu_L = \psi'(e_L)$$

$$(e_H): \mu_H = \psi'(e_H)$$

Second best:

- $\mu_H e = q_L$
- $q_L - T_L - \psi\left(\frac{q_L}{\mu_H}\right) > q_L - T_L - \psi(e_L)$ since $\frac{q_L}{\mu_H} < e_L$

$$\max_{e_L, e_H, T_L, T_H} \{\pi u(\mu_L e_L - T_L - \psi(e_L)) + (1-\pi) u(\mu_H e_H - T_H - \psi(e_L))\}$$

$$\text{s.t. } \mu_H e_H - T_H - \psi(e_H) \geq \mu_L e_L - T_L - \psi\Big(\underbrace{\frac{e_L \mu_L}{\mu_H}}_{q_L}\Big)$$

$$\pi T_L + (1-\pi) T_H \geq 0$$

effort to get q_L

$$\mu_L = \psi'(e_L) + \beta(1-\pi)\left(\mu_L - \frac{\mu_L}{\mu_H}\psi'\left(\frac{\mu_L e_L}{\mu_H}\right)\right)$$

where $\beta = \frac{u_L' - u_H'}{u_L'}$ $\quad u_L < u_H \Rightarrow u_L' > u_H'$ by concavity

$\psi'\left(\frac{\mu_L e_L}{\mu_H}\right) < \psi'(e_L)$ by convexity since $\frac{\mu_L}{\mu_H} e_L < e_L$

$$\Rightarrow \psi'(e_L) < \frac{\mu_L - \beta(1-\pi)\mu_L}{1-\beta(1-\pi)\frac{\mu_L}{\mu_H}} < \mu_L$$

$\Rightarrow$ low type works too little.

Regulation: Baron-Myerson (EMA 1982)

Regulator: Doesn't know type.

Firm: Type $\beta \in \{\underline{\beta}, \overline{\beta}\}$ with ν_1 and $1-\nu_1$

- cost $c = \beta - e$. cost is verifiable
- cost of effort $\psi(e) = \frac{e^2}{2}$
- $\Delta\beta = \overline{\beta} - \underline{\beta}$
- assume $\Delta\beta < 1$

First Best:

$\min_e \{\beta - e + e^2/2\}$

$e^* = 1$ by FOC

Firm gets $\beta - \frac{1}{2}$

$P_L = \beta_L - \frac{1}{2}$

$P_H = \beta_H - \frac{1}{2}$

Second Best

Two cost levels: $\underline{c}$ and $\bar{c}$

Two payment levels $\underline{p}$ and $\bar{p}$

Government solves

$$\min \{\nu_1 \underline{p} + (1-\nu_1)\bar{p}\}$$

$$\text{s.t. i) } \underline{p} - \underline{c} - \underline{e}^2/2 \geq \bar{p} - \bar{c} - \frac{(\bar{e} - \Delta\beta)^2}{2}$$

$$\text{ii) } \bar{p} - \bar{c} - \frac{\bar{e}^2}{2} \geq 0$$

Let $\underline{s} = \underline{p} - \underline{c} = \underline{p} - \underline{\beta} + \underline{e}$

$\bar{s} = \bar{p} - \bar{c} = \bar{p} - \bar{\beta} + \bar{e}$

$$\Rightarrow \min_{\underline{e}, \bar{e}} \left\{ \nu_1 \left(\frac{\bar{e}^2}{2} + \frac{\underline{e}^2}{2} - \frac{(\bar{e} - \Delta\beta)^2}{2} \right) + (1-\nu_1)\left(\frac{\bar{e}^2}{2} - \bar{e} \right) \right\}$$

$(\underline{e})$: $\underline{e} = 1$

$(\bar{e})$: $\nu_1 \bar{e} - \nu_1(\bar{e} - \Delta\beta) + (1-\nu_1)\bar{e} - (1-\nu_1) = 0$

$$\Rightarrow \bar{e} = \frac{1 - \nu_1 - \nu_1\Delta\beta}{1-\nu_1} = 1 - \frac{\nu_1 \Delta\beta}{1-\nu_1} < 1$$

$\Rightarrow \bar{e} < e^* \Rightarrow$ not the efficient amount