Florian Ederer
E52-368 office
Office Hours: M 10:30-12:00

Thm: (Revelation Principle): Let $\Gamma = (S_1, \ldots, S_I, g(\cdot))$ implement scf $f(\cdot)$ in dominant strategies. Then $f(\cdot)$ is truthfully implementable in dominant strategies.

Proof: Since Γ exists, there exists $s^*(\cdot) = (s_1^*(\cdot), \ldots, s_I^*(\cdot))$ such that $g(s^*(\theta)) = f(\theta) \quad \forall \theta \in \Theta = \Theta_1 \times \ldots \times \Theta_I$

It must be that

$$u_i(g(s_i^*(\theta_i), s_{-i}(\theta_{-i})) \geq u_i(g(\hat{s}_i(\theta_i), s_{-i}(\theta_{-i}))$$

$$\forall \hat{s}_i \in S_i \quad \forall \theta_i \in \Theta_i \quad \forall s_{-i} \in S_{-i} = \times_{j \neq i} S_j$$

$\forall \theta_i \in \Theta_i$, we must have that

$$u_i(g(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})); \theta_i)$$
$$\geq u_i(g(s_i(\hat{\theta}_i), s_{-i}(\theta_{-i})); \theta_i)$$

Since $g(s^*(\theta)) = f(\theta)$, we have for a fixed $\theta \in \Theta$

$$u_i(f(\theta_i, \theta_{-i}); \theta_i) \geq u_i(f(\hat{\theta}_i, \theta_{-i}); \theta_i)$$

$\forall \hat{\theta}_i \in \Theta_i$. $\square$

Gibbard-Satterthwaite: If $R_i = P \quad \forall i$, $f(\Theta) = X$, X has at least three elements, then the scf f is truthfully implementable iff f is dictatorial.

Sketch of proof:

1] If $R_i = P$ $\forall i$, for $f(\cdot)$ to be truthfully implementable, f must be monotonic.

2] Then $f(\cdot)$ is ex post efficient

3] If $f(\cdot)$ is monotonic and efficient, f is dictatorial.

The important aspect is that we can get around this theorem. 1] Relax $R_i = P$ to look at only quasilinear preferences, or 2] Look at BNE implementation instead of dominant strategy implementation.

(weak) implementation ⇒ One equilibrium is desirable

(strong) implementation ⇒ all equilibria get you what you want.

- The problem of multiplicity is important, but it is usually sidestepped.

Next week: Weaknesses of revelation principle in the Bayesian Nash equilibrium implementation.