Schedule appointment for OH
4 problem sets. 1st is assigned on Sept. 19th.
Grade = 0.2 PS + 0.8 Final
Can work in groups. Write-up separately.
Final exam: December 12th in class: 4-6:30
  └ easier version of problem sets.
Read as many starred papers as possible.
Purchase Bolton/Dewatripont.
Lecture notes will be available at the end of the week

Competitive markets - GE
Strategic situations (small numbers) - Game theory
Small numbers where this some element of design.
- Game theory with endogenous rules.
- Mechanism design.

Insurance
2 parties: A, B
$I_A = (0, \frac{1}{3}; 100, \frac{1}{3}; 200, \frac{1}{3})$
A is risk averse, B is risk neutral.
If A has all the bargaining power, B will assume all the risk - pay A \$100. First best is full insurance.
Perhaps A can influence the probabilities (hidden actions - moral hazard)
There might be adverse selection
Both of these will prevent first best outcome.

A- Electricity plant
B- Coal mine
Electricity plant costs 100
Electricity plant purchases coal.
Revenue is 180 if plant purchases coal.
Cost of coal is 20
Since 180-100-20=60>0, it is efficient to trade
Suppose Nash bargaining in period 2: price will be 100
$\Pi_{\text{Electricity}} = 180-100-100=-20 < 0 \Rightarrow$ not enter
There will be inefficiencies. Three solutions.
- Vertical integration
- Long-term contract
- Allocation of bargaining power

<u>Mechanism Design</u>

What constraints are there on information that can be elicited?

I agents $i=1,\ldots,I$
Collective choice $x \in X$
Each agent privately observes $\theta_i \in \Theta_i$
Bernoulli utility function $u_i(x, \theta_i)$
$\succsim_i(\theta_i)$ ordinal preference ranking on X (depends on type.)
Common prior φ over distribution of types:
ie. φ is a prob. measure over $\Theta \equiv \Theta_1 \times \ldots \times \Theta_I$
- Harsanyi doctrine - common prior.

Defn: A <u>social choice function</u> is a map $f : \Theta \to X$.

Defn: f is <u>ex post efficient</u> if $\nexists$ a profile $\theta = (\theta_1, \ldots, \theta_I)$ in which $\exists$ any choice $x \in X$ satisfying

$u_i(x; \theta_i) \geq u_i(f(\theta); \theta_i)$ $\forall i$

$u_i(x; \theta_i) > u_i(f(\theta); \theta_i)$ some i.

- Selects the choice that is Pareto Efficient for a given profile $\theta \in \Theta$.

Defn: a <u>mechanism</u> $\Gamma = (S_1, \ldots, S_I, g(\cdot))$ is an $I+1$-tuple consisting of a strategy set S_i for each individual and an <u>outcome function</u> $g : S_1 \times \ldots \times S_I \to X$.

Eg auctions:

$S_i = \mathbb{R}_+$ $i = 1, \ldots, I$

$g = (b_1, \ldots, b_I) = (\{y_i(\cdot)\}_{i=1}^I, \{t_i(\cdot)\}_{i=1}^I)$

$y_i = 1$ if $b_i \geq b_{i'}$ $\forall i' \neq i$

$t_i = b_i$, $t_{i'} = 0$

Our definition of a mechanism is related to normal form games.

Defn: a <u>strategy</u> is a function $s_i : \Theta_i \to S_i$

Defn: The mechanism Γ implements a social choice function f if there exists equilibrium strategies $s_1^*(\theta_1), \ldots, s_I^*(\theta_I)$ of the game G such that $g(s_1^*(\theta_1), \ldots, s_I^*(\theta_I)) = f(\theta_1, \ldots, \theta_I)$ for all $(\theta_1, \ldots, \theta_I) \in \Theta$

Here $G = (\Theta_1, \ldots, \Theta_I; u_1, \ldots, u_I; \varphi_1, \ldots, \varphi_I; S_1, \ldots, S_I)$ is the Bayesian game corresponding to the mechanism Γ.

What do we mean by "equilibrium"? What do we do if there is more than one equilibrium.

Two approaches:

- dominant strategy equilibrium
- BNE

Defn: a direct revelation mechanism is a mechanism in which $S_i = \Theta_i \; \forall i$. In addition, $g(\theta) = f(\theta)$ $\forall \theta \in \Theta$.

Defn: The social choice function f is incentive compatible if the direct revelation mechanism Γ has an equilibrium $(s_1^*(\theta_1), \ldots, s_I^*(\theta_I))$ in which $s_i^*(\theta_i) = \theta_i \; \forall \theta_i \in \Theta_i$ and $\forall i$

Until further notice, we will be dealing with dominant strategy equilibria.

$u_i(f(\theta_i, \theta_{-i}); \theta_i) \geq u_i(f(\hat{\theta}_i, \theta_{-i}); \theta_i) \quad \forall \hat{\theta}_i \in \Theta_i \ \forall i$

implies that truth-telling is a <u>dominant strategy equilibrium</u>.

$\mathcal{P} = \{ \succsim \subset X \times X : \succsim$ is complete and transitive$\}$ (rational)

$\mathcal{R}_i = \{ \succsim_i \subset X \times X : \succsim_i$ is rational$\}$

$f(\Theta) = \{x \in X : f(\theta) = x$ for some $\theta \in \Theta\}$

<u>Defn</u>: f is <u>dictatorial</u> if $\exists\, i$ s.t. $\forall\, \theta \in \Theta$ we have

$$f(\theta) \in \{x \in X : u_i(x, \theta_i) \geq u_i(y, \theta_i) \ \forall y \in X\}.$$

<u>Theorem</u>: (Gibbard-Satterthwaite) Suppose

i) X is finite and contains at least three elements

ii) $\mathcal{R}_i = \mathcal{P} \ \forall i$

iii) $f(\Theta) = X$

Then the scf f is dominant strategy implementable iff f is dictatorial.

Can weaken

i) dominant strategy implementation

ii) $\mathcal{R}_i \subset \mathcal{P}$

Quasilinear preferences

$x \in X \Rightarrow x = (k, t_1, \ldots, t_I)$ $k \in K$ is a "project"
t_i is a monetary transfer to i.

$u_i(x;\theta) = v(k;\theta) + (\bar{m}_i + t_i)$ where $\bar{m}_i$ is i's endowment of money

$X \subseteq K \times \mathbb{R}^I = \{(k, t_1, \ldots, t_I) : k \in K, t_i \in \mathbb{R}\ \forall i, \sum_{i=1}^{I} t_i \leq 0\}$

announce $\theta_i \longrightarrow k \in K \longrightarrow u_i, u_{-i}$

t_i should depend on the externality i imposes on $-i$.

Prop: Let $k^*(\cdot)$ be a function which is ex-post efficient. The scf f is truthfully implementable in dominant strategies if $\forall i$

$$t_i(\theta) = \sum_{j \neq i} v_j(k^*(\theta)) + h(\theta_{-i})$$

(Groves-Clark mechanism)

Next time: Bayesian implementation