14.271: Industrial Organization I

Notes on Monopoly Pricing

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1 Standard Single Product Monopoly Pricing

Suppose a firm has a cost function $c(q)$ and faces an inverse demand curve $p(q)$. The firm then wants to

$$\max_q qp(q) - c(q).$$

Taking first order conditions, we have

$$(q) : p(q) + qp'(q) - c'(q) = 0$$

or

$$\begin{aligned} p(q) - c'(q) &= -qp'(q) \\ \frac{p(q) - c'(q)}{p(q)} &= -\frac{1}{\frac{p(q)}{qp'(q)}} = -\frac{1}{\varepsilon}, \end{aligned}$$

where $\varepsilon = \frac{dp}{dq}\frac{q}{p}$ is the elasticity of demand. (It will be negative, since $\frac{dp}{dq} < 0$.) We refer to the quantity $-\frac{1}{\varepsilon}$ as the Lerner index, and note that it is a measure of a percentage markup over marginal cost when a firm faces a downward sloping demand curve.

Under perfect competition, we have that $\varepsilon = -\infty$, so we have that $p(q) = c'(q)$, or marginal cost pricing. If $\varepsilon > -1$, (i.e. $|\varepsilon| < 1$) then we have that

$$\begin{aligned} \frac{p(q) - c'(q)}{p(q)} &> 1 \\ p(q) - c'(q) &> p(1) \\ -c'(q) &> 0 \\ c'(q) &< 0, \end{aligned}$$

which has no solution (assuming positive marginal costs). Thus, a monopolist will not price on a part of the demand curve in which $\varepsilon > -1$.

2 Multi-Product Monopoly Pricing

Suppose a firm can produce N commodities with demand curves given by $D_i(p_1, \ldots, p_N) = D_i(p)$ for $i = 1, \ldots, N$. Further, suppose the firm's cost is given by $C(D_1(p), \ldots, D_N(p)) = C(D)$. The monopolist then wants to

$$\max_{p_1,\ldots,p_N} \sum_{i=1}^{N} p_i D_i(p) - C(D_1(p), \ldots, D_N(p)).$$

Taking first order conditions, we have

$$(p_i) : \underbrace{D_i(p) + p_i \frac{\partial D_i(p)}{\partial p_i}}_{MR \text{ from product } i} + \underbrace{\sum_{j \neq i} p_j D_j(p)}_{MR \text{ from products } -i} = \underbrace{\sum_{i=1}^{N} \frac{\partial C(D)}{\partial D_i} \frac{\partial D_i(p)}{\partial p_i}}_{MC}.$$

2.1 Separable Cost Functions

In order to get some intuition for this result, suppose $C(D_1(p), \ldots, D_N(p)) = \sum_{i=1}^{N} C_i(D_i(p))$. The first order conditions then become

$$(p_i): 0 = D_i(p) + p_i \frac{\partial D_i(p)}{\partial p_i} + \sum_{j \neq i} p_j \frac{\partial D_j(p)}{\partial p_i} = \underbrace{\frac{\partial C_i(D_i)}{\partial D_i}}_{\equiv C_i'} \frac{\partial D_i(p)}{\partial p_i} + \sum_{j \neq i} \underbrace{\frac{\partial C_j(D_j)}{\partial D_j}}_{\equiv C_j'} \frac{\partial D_j(p)}{\partial p_i}.$$

Rearranging,

$$\begin{aligned}
(p_i - C_i') \frac{\partial D_i(p)}{\partial p_i} &= -D_i(p) - \sum_{j \neq i} \left[\left(p_j - C_j'\right) \frac{\partial D_j(p)}{\partial p_i} \right] \\
\frac{p_i - C_i'}{p_i} &= -\frac{1}{\frac{p_i}{D_i(p)} \frac{\partial D_i(p)}{p_i}} - \sum_{j \neq i} \left[\frac{p_j - C_j'}{p_i} \frac{\frac{\partial D_j(p)}{\partial p_i}}{\frac{\partial D_i(p)}{\partial p_i}} \right] \\
&= -\frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \left[\frac{p_j - C_j'}{p_i D_i} \frac{D_j(p) \frac{p_i}{D_j(p)} \frac{\partial D_j(p)}{\partial p_i}}{\frac{p_i}{D_i} \frac{\partial D_i(p)}{\partial p_i}} \right] \\
&= -\frac{1}{\varepsilon_{ii}} - \sum_{j \neq i} \left[\frac{\left(p_j - C_j'\right) D_j(p) \varepsilon_{ij}}{R_i \varepsilon_{ii}} \right],
\end{aligned}$$

where

$$\varepsilon_{ii} \equiv \frac{\partial D_i(p)}{\partial p_i} \frac{p_i}{D_i(p)} \text{ and } \varepsilon_{ij} = \frac{\partial D_j(p)}{\partial p_i} \frac{p_i}{D_j(p)}.$$

Thus, we have that

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\varepsilon_{ii}} - K.$$

If $K < 0$, then the firm's markup for product i will be greater than it would be if product i were the only product the firm produced. $K < 0$ results when "enough" of the other products that the firm offers (weighted by their demand) are substitutes for product i. Similarly, if $K > 0$, then the firm's markup for product i will be less than it would be if product i were its only product. In particular, if $K > -\frac{1}{\varepsilon_{ii}}$, then the firm will price commodity i below cost.

2.2 Two Product Case

To develop some more intuition for the above result, suppose $N = 2$. Then our monopoly pricing rule for commodity 1 is given by

$$\begin{aligned}
\frac{p_1 - C_1'}{p_1} &= -\frac{1}{\varepsilon_{11}} - \frac{(p_2 - C_2') D_2(p) \varepsilon_{12}}{R_1 \varepsilon_{11}} \\
&= -\frac{1}{\varepsilon_{11}} - K.
\end{aligned}$$

Clearly, it cannot be the case that both $p_1 - C_1' < 0$ and $p_2 - C_2' < 0$. (If the firm sold both of its products at below cost, then it would be making negative profits!) Without loss of generality, assume $p_2 - C_2' > 0$ so that analyzing $\frac{p_1 - C_1'}{p_1}$ gives intuitive results. (If $p_2 - C_2' < 0$, then the expression for $\frac{p_2 - C_2'}{p_2}$ would be the one which would supply us these intuitive results.) Since $p_2 - C_2' > 0$, $D_2(p) > 0$, and $R_1 \varepsilon_{11} > 0$, we have that $K < 0$ iff $\varepsilon_{12} < 0$, which is equivalent to $\frac{\partial D_2(p)}{\partial p_1} \frac{p_1}{D_2(p)} < 0$, or $\frac{\partial D_2(p)}{\partial p_1} < 0$. (i.e. commodities 1 and 2 are gross substitutes.) Similarly, $K > 0$ if and only if $\frac{\partial D_2(p)}{\partial p_1} > 0$. (i.e. commodities 1 and 2 are gross complements.)

3 Product Quality

Suppose we have a continuum of consumers of unit mass with types $\theta \sim U[0,1]$. A consumer of type θ has utility

$$u(\theta) = \begin{cases} v(s,\theta) - p & \text{if buys 1 unit of quality } s \text{ at price } p \\ 0 & \text{if not buy} \end{cases},$$

where $\frac{\partial v}{\partial s} > 0$, $\frac{\partial v}{\partial \theta} > 0$. The firm has constant marginal cost $c(q,s) = c(s)$ that is a function only of the quality of the good provided. Suppose the monopolist chooses to produce at quality s. For a given price p, we will have that for all θ such that $v(s;\theta) - p > 0$, the consumer will buy. Let $\hat{\theta}$ solve $v\left(s,\hat{\theta}\right) = p$. Then all consumers with $\theta \geq \hat{\theta}$ will purchase the good. That is, the firm will sell $1 - \hat{\theta} = q$ units. (From which we can see that $\hat{\theta} = 1 - q$.) Thus, the firm's problem is to

$$\max_{q,s} q\left[v(s, 1-q) - c(s)\right].$$

Let us examine how the monopolist's choice of quality compares to the social optimum. Taking first order conditions with respect to quality, we have

$$(s): \frac{\partial v(s^m, 1-q^m)}{\partial s} = \frac{\partial c(s^m)}{\partial s}.$$

The social planner, on the other hand, wants to

$$\begin{aligned} &\max_{q,s} \pi(q,s) + CS(q,s) \\ = &\max_{q,s} \int_{1-q}^{1} \left[v(s,\theta) - p + p - c(s)\right] d\theta \text{ (integrating over all types that buy)} \\ = &\max_{q,s} \int_{1-q}^{1} \left(v(s,\theta) - c(s)\right) d\theta = \max_{q,s} \int_{1-q}^{1} v(s,\theta)\, d\theta - qc(s) \end{aligned}$$

Taking first order conditions with respect to quality, we have

$$(s): \int_{1-q}^{1} \frac{\partial v\left(s^{FB},\theta\right)}{\partial s} d\theta - q\frac{\partial c\left(s^{FB}\right)}{\partial s} = 0$$

or

$$\frac{\partial c\left(s^{FB}\right)}{\partial s} = \frac{1}{q}\int_{1-q}^{1} \frac{\partial v\left(s^{FB},\theta\right)}{\partial s} d\theta.$$

Comparing this FOC to the monopolist's FOC, we have

$$\frac{\partial c(s^m)}{\partial s} = \frac{\partial v(s^m, 1-q^m)}{\partial s}.$$

Here, we note that we are evaluating the monopolist's FOC at $\hat{\theta} = 1 - q^m$. That is, we are concerned with only the marginal utility of quality for the marginal consumer. In contrast, we evaluate the societal first order condition at $\frac{1}{q}\int_{1-q}^{1} \frac{\partial v\left(s^{FB},\theta\right)}{\partial s} d\theta$. That is, we are concerned with the average of the marginal utility of quality for those consumers who buy. (The $\frac{1}{q}$ factor at the beginning normalizes for the fact that we see only q units.)

If $\frac{\partial v(s,\theta)}{\partial s} = k$ for all θ, then we have that

$$\frac{\partial c\left(s^{FB}\right)}{\partial s} = \frac{1}{q}\int_{1-q}^{1} \frac{\partial v\left(s^{FB},\theta\right)}{\partial s} d\theta = \frac{1}{q}\int_{1-q}^{1} k\, d\theta = \frac{k\left(1-(1-q)\right)}{q} = k,$$

but $k = \frac{\partial v(s, 1-q^m)}{\partial s}$ as well. Thus,

$$\frac{\partial c\left(s^{FB}\right)}{\partial s} = \frac{\partial v\left(s^{FB}, 1-q^m\right)}{\partial s},$$

and we see that $s^{FB} = s^m$.