Signalling - Q2
- costly action to signal private information
- Milgrom, Roberts '82
- Spence: "Job Signalling"

Signal Jamming
- attempt to influence the distribution of some commonly observed signal
- Fudenberg / Tirole
- Holmstrom, Career Concerns
- "rat race"

"Fighting"
- chain store paradox
- repeated games

Auctions

"Direct revelation mechanisms"
- N bidders, $x_i \sim F_i[0, w_i]$
- mechanism: (B, Π, μ)
  - B_i - set of bids for i
  - $\Pi: B \to \Delta$ allocation rule
  - $\mu: B \to \mathbb{R}^N$ payment rule
- Focus on (Q, M) direct mechanism
  - $B_i = X_i \quad \forall i$
  - $Q: X \to \Delta \qquad Q_i(x) =$ prob. of winning
  - $M: X \to \mathbb{R}^N \qquad M_i(x) =$ payment
- $q_i(\underbrace{z_i}) = \int_{x_{-i}} Q_i(z_i, x_{-i}) f_{-i}(x_{-i})\, dx_{-i}$
  - announcement
  - prob of winning given announcement z_i

- $m_i(z_i) = \int_{x_{-i}} M_i(z_i, x_{-i}) f_{-i}(x_{-i})\, dx_{-i}$
  - expected payment
- $U_i(x_i) = q_i(z_i)\, x_i - m_i(z_i)$

  IC $\rightarrow$ $U_i'(x_i) = q_i(z_i)$

  $\Rightarrow U_i(x_i) = U_i(0) + \int_0^{x_i} q_i(t_i)\, dt_i$

  $\Rightarrow m_i(x_i) = q_i(x_i)\, x_i - U_i(x_i)$

For IPV, symmetric, risk neutral players, $q_i(x_i)$ is the same, and hence in expectation, we have revenue equivalence.

Optimal auctions

Seller wants to $\max E[R] = \max \sum_{i=1}^{N} E[m_i(x_i)]$

- $m_i(x_i) = q_i(x_i)\, x_i - U_i(x_i)$

  $= q_i(x_i)\, x_i - U_i(0) - \int_0^{x_i} q_i(t_i)\, dt_i$

Integration by Parts yields

$$E[m(x_i)] = \int_0^{w_i} \underbrace{\left(x_i - \frac{1 - F_i(x_i)}{f_i(x_i)}\right)}_{\equiv \varphi_i \,=\, \text{virtual valuation}} q_i(x_i) f_i(x_i)\, dx_i$$

- need to pay informational rents for type to be revealed

$$\Rightarrow \max \int_x \left(\sum_{i=1}^{N} \varphi_i(x_i) Q_i(x) \right) f(x)\, dx$$

with $M_i(x_i) = Q_i(x)x_i - \int_0^{x_i} Q_i(z_i, x_{-i})\, dz_i$

- In general, revenue maximizing auction need not be socially optimal. This is because we can view this as a monopoly problem.

Departures:

Risk aversion $\Rightarrow E[R^{FP}] > E[R^{SP}]$

- Let $\gamma(x)$ be the bidding function under risk aversion
- SP: bidding is still the same
- FP: $\max G(z)u(x-\gamma(z))$

$$\Rightarrow \text{FOC: } g(z)u(x-\gamma(z)) - G(z)\gamma'(z)u'(x-\gamma(z)) = 0$$

$\Rightarrow$ In symmetric equilibrium, will have $z = x$

$$\Rightarrow \frac{g(x)u(x-\gamma(x))}{\gamma'(x)} = G(x)u'(x-\gamma(x))$$

$$\Rightarrow \gamma'(x) = \frac{u(x-\gamma(x))}{u'(x-\gamma(x))} \frac{g(x)}{G(x)}$$

In risk neutral case, $\beta'(x) = (x-\beta(x))\dfrac{g(x)}{G(x)}$

$u(0)=0;\ u''<0 \Rightarrow \forall y,\ \frac{u(y)}{u'(y)} > y$

$\Rightarrow \gamma'(x) > \beta'(x)$, and $\gamma(0) \geq \beta(0)$

$\Rightarrow$ FP will generate more revenue

(ii) asymmetric Bidders: $[0, w_i]$

- Two bidders with $F_1[0, w_1]$ and $F_2[0, w_2]$
  - Let $\varphi_1 = \beta_1^{-1}$, $\varphi_2 = \beta_2^{-1}$
  - $\pi_i(b, x_i) = F_j(\varphi_j(b))(x_i - b)$
    $= H_j(b)(x_i - b)$

$\Rightarrow$ FOCs: $h_j(b)(\varphi_i(b) - b) = H_j(b)$

$$\Leftrightarrow \quad \varphi_j'(b) = \frac{F_j(\varphi_j(b))}{f_j(\varphi_j(b))(\varphi_i(b) - b)}$$

- assume $b(0)=0$ and $b(w_i) = \bar{b}$

assume $\frac{f_1(x)}{F_1(x)} > \frac{f_2(x)}{F_2(x)} \Rightarrow \beta_1(x) < \beta_2(x)$

- weaker guy bids more aggressively.