Long Purse Theory

$t=1$	$t=2$	$t=3$
Firms 1, 2 compete ∘ $\pi_1(a_1, a_2)$ ∘ $\pi_2(a_1, a_2)$	Firm 2 borrows $K-E$ or exits $E = \pi_2(a_1, a_2)$ ∘ retained earnings	Monopoly or duopoly

Idea: Lower prices at $t=1$
- $\Rightarrow$ E smaller
- $\Rightarrow$ Borrowing more costly
- $\Rightarrow$ May be possible to force exit.

This story will not make sense if there are perfect capital markets.

Debt contract in Imperfect Capital Markets

Stage 1: Firm has capital E. Needs K to undertake investment with random return $\pi \overset{cdf}{\sim} F$
- ∘ Firm can borrow $K-E$ at interest rate r.

Stage 2: Firm gets π. Has two choices:

	U_{firm}	U_{Bank}
Repay	$\pi-(1+r)(K-E)$	$(1+r)(K-E)$
Default	0	$\pi - \underbrace{a}_{\text{socially wasteful bankruptcy cost}}$

- ∘ Define $R=(1+r)(K-E)$ to be the repayment
- ∘ Bank's profit is:

$$\pi_B(R,E) = (1-F(R))R + \int_0^R \pi \, dF(\pi) - aF(R) - (K-E)$$

- Let $R^*(E)$ be the smallest solution to $\Pi_B(R^*(E), E)) = 0$

Observations:

1] $\frac{dR^*}{dE} < 0$ (ie. borrow less ⇒ repay less)

- $$\frac{dR}{dE} = -\frac{\overbrace{\frac{d\pi}{dE}}^{>0}}{\underbrace{\frac{d\pi}{dR}}_{>0}} < 0$$ by implicit function thm.

2] Firm profits: $E[\pi - E - \text{Repayments}] \uparrow$ in E

- $E[\text{Repayment}] = aF(R^*) + (K - E)$
- $\underbrace{E[\pi - E - \text{Repayment}]}_{\equiv M} = (\pi - E) - (K-E) - aF(R^*)$
$= \pi - K - aF(R^*(E))$

$$\Rightarrow \frac{dM}{dE} = -\underbrace{af(R^*(E))}_{>0}\underbrace{\frac{dR^*}{dE}}_{<0} > 0$$

Thus, E small ⇒ optimal for firm to exit.

3] Maximum borrowing limit is less than expected profits.

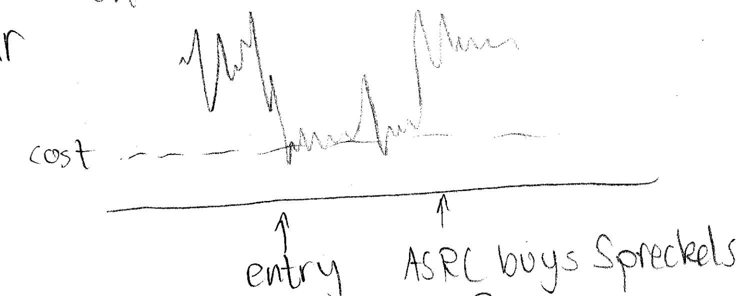
- Reducing E makes the firm involuntarily decide not to continue.

Genesove and Mullin "Predation and its Rate of Return: The Sugar Industry $\underbrace{1887}_{\text{Sherman act passed}}$-1914."

1887: Sugar Trust / ASRC (american Sugar Refining Company)

- capacity 35 K barrels/day. 85% of Eastern US capacit

1889: Spreckels Sr. entered at 6K barrels/day
- price war



- ASRC loses ≈12M over 2 years

1892: ASRC buys Spreckels - now controls 95%
- of capacity

1892-1897: Three smaller entrants ≈ 1300 barrels per day. No price war.

1898: Arbuckle / Doscher - 3K barrels per day each. Price war. Looks just like previous graph. Arbuckle, Doscher form NSRC under control of ASRC.
- ASRC + NSRC ≈ 87%

1901-1910: More small scale entry. Share ≈ 80%

1910: Antitrust suit - ASRC + NSRC break up

Observations:

1. Prices should have been very high.
   - Margin should have been ≈ 75¢ under residual demand model
   - Prices are ≈ 50¢ under Bertrand. (w/cap. constraints
2. Prices seem too low, even before Arbuckle - Doscher war.

3] Price wars took the summer off. (Canning process)

4] Predation appears to have lowered buyout prices.

- Doscher got about 400K per year
  - at pre-entry prices, profits would have been about 4.5 M/year.
- Loss during the price war was ≈ 21M for the ASRC.

Comments on Theories

1] Cost-signalling to lower buyout prices seems implausible.

2] Find no evidence of financing constraints.

3] Suggests more complex reputation story signalling future conduct.

- fight big entrants, take summers off, low buyout prices.

4] Reputation had entry deterrence benefits.

Auctions:

Independent Private Values:

- N bidders
- Values $\{v_1, \dots, v_N\}$ $v_i \overset{iid}{\sim} F$. Often $U[0,1]$
- $\text{utility}_i = \begin{cases} v_i - p & \text{if wins at } p \\ 0 & \text{if loses} \end{cases}$

- assume v_i is private information
  - distribution is common knowledge

English auction:

- Suppose b increases in continuous time and people need to keep their hand up. Last person with hand up wins.
- Equilibrium strategies: drop out at v_i.
- $E[\text{Revenue}] = \int_0^1 v \underbrace{N(N-1)(1-F(v))f(v)F(v)^{N-2}}_{\text{second order statistic}} dv$
  - v is winning bid if 1 person has $v_i > v$, $N-2$ have $v_j < v$, and one person has $v_k = v$

$\Rightarrow E[\text{Revenue}] = \frac{N-1}{N+1}$ for $v_i \sim U[0,1]$

- alternatively, could look at second price sealed bid auction:
  - write down bid
  - high bid wins.
  - pays second highest bid.
- This makes this a simultaneous move game.

First-price sealed bid auction

- Highest bidder wins and pays bid

Observation 1: Bidding $b_i = v_i$ is a bad idea.

- $\pi(b_i, b^*_{-i}; v_i) = (v_i - b_i) \Pr[b_i \geq \max_{j \neq i} b_j(v_j)]$

$= (v_i - b_i) \prod_{j \neq i} \Pr[b_j^*(v_j) \leq b_i]$

$= (v_i - b_i) \prod \Pr[v_j \leq b_j^{*-1}(b_i)]$

$$= (v_i - b_i) \prod_{j \neq i} F(b_j^{*-1}(b_i))$$

Now look for a symmetric BNE

$$\max_{b_i} \Pi(b_i, b_{-i}^*; v_i) = \max_{b_i} (v_i - b_i) F(b^{*-1}(b_i))^{N-1}$$

FOC:

$$(b_i): (v_i - b_i) \cdot (N-1) F(b^{*-1}(b_i))^{N-2} f(b^{*-1}(b_i)) \frac{db^{*-1}}{db_i} - F(b^{*-1}(b_i))^{N-1} = 0$$

$$\Rightarrow (v_i - b_i)(N-1) f(b^{*-1}(b_i)) \frac{db^{*-1}}{db_i} = F(b^{*-1}(b_i))$$

$$\Rightarrow v_i - b_i = \left. \frac{F(b^{*-1}(b_i))}{(N-1) f(b^{*-1}(b_i)) \frac{db^{*-1}}{db_i}} \right|_{b_i = b^*(v_i)}$$

$$\Rightarrow v_i - b^*(v_i) = \underbrace{\frac{F(v_i)}{(N-1) f(v_i)} \cdot \frac{db^*}{dv_i}}$$

analogous to monopoly markup

- differential equation

For $v_i \sim U[0,1]$, $b^*(v) = \frac{N-1}{N} v$

$$E[\text{Revenue}] = \int_0^1 \left(\frac{N-1}{N} v\right) N f(v) (1 - F(v))^{N-1} dv$$

$$= \frac{N-1}{N+1}.$$

- same as in English auction.

Next time: Revenue equivalence and departures.