In IV, R^2 is meaningless

Conduct Estimation: $\frac{p-c'}{p} = \frac{\theta}{|\varepsilon|}$, θ is a measure of collusion

$\theta \in [0,1]$. $\theta \approx 1 \Rightarrow$ almost perfect collusion

$\theta \approx 0 \Rightarrow$ competitive.

Porter gets identification from Δregime in industry

Perfect Public Equilibrium

- Cournot competition
- N firms, symmetric
- $\pi_i(p, q_i)$
- $p_t = \theta_t \, p(\sum_{i=1}^{N} q_{it})$ $\quad \theta \overset{iid}{\sim} F(\theta)$
- trigger price $\bar{p}$ s.t. if $p_t < \bar{p}$, play Cournot for T periods. If $p_t \geq \bar{p}$, continue colluding.
  - Cournot: q^c
  - colluding: q^t
- Because of uncertainty, there is no proper subgame so SPE has no bite.
- Here, strategies are dependent only on public signals: $p_1, p_2, \ldots$
- $V_i(q_i)$, $\pi_i(q_i) = E_\theta[\pi_i(q_i, \theta_t p(q_i + q_{-i}))]$

$\pi_i(q^c) = E_\theta[\pi_i(q^c, \theta_t p(nq^c))]$

$$V_i(q_i) = \Pi_i(q_i) + \delta[\Pr[\bar{p} \le \theta p(q_i+q_{-i})] V_i(q_i)]$$
$$+ \delta[\Pr[\bar{p} > \theta p(q_i+q_{-i})] \left(\frac{1-\delta^T}{1-\delta}\Pi_i(q^c) + \frac{\delta^T}{1-\delta} V_i(q_i)\right)]$$

Note: $\Pr[\bar{p} > \theta p(q_i+q_{-i})] = F\left(\frac{\bar{p}}{p(q_i+q_{-i})}\right)$

$$\Rightarrow V_i(q_i) = \underbrace{\frac{\Pi_i(q^c)}{1-\delta}}_{\text{base profit}} + \underbrace{\frac{\Pi_i(q_i) - \Pi_i(q^c)}{1-\delta(1-F(\cdot)) - \delta^{T+1}F(\cdot)}}_{\text{extra profit from colluding}}$$

Is q_i enforceable?

The FOC is:
$$\overbrace{\Pi_i'(q_i)[1-\delta(1-F(\cdot)) - \delta^{T+1}F(\cdot)]}^{\text{gain from cheating}}$$
$$+ \underbrace{\delta[\Pi_i(q_i) - \Pi_i(q^c)](1-\delta^T)}_{\text{loss if punished}} \underbrace{f(\cdot)\frac{\bar{p}\,p(\cdot)}{p(\cdot)^2}}_{\text{prob of punishment}}$$

- What is $\bar{p}, T, q_i$?
- In general, $q^t > q^M$

Note that if $C''(q) > 0$, Bertrand competition does not lead to $p = MC$. Rather, it leads to the use of mixed strategies.

Rotemberg ~~and Saloner~~

Everything is observed

- $P(Q_t, \varepsilon_t)$, $\varepsilon_t \overset{iid}{\sim} F(\varepsilon)$, $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$, $P_\varepsilon(Q_t, \varepsilon_t) = \frac{\partial P}{\partial \varepsilon_t} > 0$
- N firms, Bertrand, $MC = c$
- $\Pi^m(\varepsilon_t)$ fully collusive total joint maximizing profit
  - firm i if collude: $\frac{\Pi^m(\varepsilon_t)}{N}$ versus $\Pi^m(\varepsilon_t)$ if slightly undercut
- Let K = continuation value
- Will not deviate when ε_t if:

$$\frac{\Pi^m(\varepsilon_t)}{N} + K \geq \Pi^m(\varepsilon_t) \iff \Pi^m(\varepsilon_t) \leq \frac{KN}{N-1}$$

- will never deviate if $\Pi^m(\bar{\varepsilon}) \leq \frac{KN}{N-1}$
- otherwise, $\exists\, \varepsilon^*$ s.t. $\Pi^m(\varepsilon^*) = \frac{KN}{N-1}$
  - for $\varepsilon_t > \varepsilon^*$, cannot get Π-max choice
    - Let $\Pi^s(\varepsilon_t, \varepsilon^*)$ be per firm sustainable Π.
    - $\Pi^s(\varepsilon_t, \varepsilon^*) = \begin{cases} \frac{\Pi^m(\varepsilon_t)}{N} & \varepsilon_t \leq \varepsilon^* \\ \frac{K}{N-1} & \varepsilon_t \geq \varepsilon^* \end{cases}$

$$\Rightarrow K(\varepsilon^*) = \frac{\delta}{1-\delta}\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \pi^s[\varepsilon_t, \varepsilon^*]\,dF(\varepsilon)$$

$$= \frac{\delta}{1-\delta}\left[\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \pi^m(\varepsilon_t)\,dF(\varepsilon) + (1-F(\varepsilon^*))\,\pi^m(\varepsilon^*)\right]$$

$$\Rightarrow \max K(\varepsilon^*) \text{ s.t. } \pi^m(\varepsilon^*) = \frac{K(\varepsilon^*)}{N-1}$$

- ie want to make continuation value as high as possible while still mantaining incentives not to cheat.