Dynamic Competition

- Repeated Bertrand competition
  - N firms
  - $D(p)$ demand curve
  - Choose prices $p_1, \ldots, p_N$ at $t = 0, 1, 2, \ldots$
  - For $\delta \geq 1 - \frac{1}{N}$, $\exists$ SPE with

$$p_{it}^*(h_t) = \begin{cases} p^m & \text{if everyone charged } p^m \text{ in the past} \\ c & \text{otherwise} \end{cases}$$

  - For $\delta < 1 - \frac{1}{N}$, $p_{it}^*(h_t) = c$
    - collusion ought to be very easy

(*) are there other ways of looking at repeated competition that do not yield this folk theorem result?

(*) Can we model this in such a way that as N gets larger, it gets more difficult to collude?

Imperfect Monitoring (Green, Porter)

- 2 firms, Bertrand competition
  - CRS, cost c
- Noisy demand: $D_t(p_t) = \begin{cases} D(p) & \text{w/prob } 1-\alpha \\ 0 & \text{w/prob } \alpha \end{cases}$
- assume that firms observe their own demand but not rival's demand.
- Possible collusive equilibrium:

- Charge p^m until I sell nothing. Then charge c for T periods and then go back to charging p^m.

<u>Proposition:</u> For $\alpha < \frac{1}{2}$ the strategy above is a SPE if δ is close to 1 and T is large enough.

<u>Proof:</u> [1] Subgames with $p_{it}^*(h_t) = c$

- gain in 1st pd from deviating: 0
- long-run effect of deviating: 0

[2] Subgames with $p_{it}^*(h_t) = p^m$

- $V^m \equiv$ PDV of following the strategy in cooperative phase

  $V^p \equiv$ PDV of following the strategy at the start of the punishment phase

- If follow strategies, get V^m.
- If deviate: $\pi^m(1-\alpha) + \delta V^p$
  - Need to show that $V^m \geq \pi^m(1-\alpha) + \delta V^p$
  - To find those:

$$V^m = \frac{\pi^m}{2}(1-\alpha) + \delta V^m (1-\alpha) + \alpha \delta V^p$$

$$V^p = \delta^T V^m$$

$$\Rightarrow V^m = \frac{\frac{\pi^m}{2}(1-\alpha)}{1-\delta(1-\alpha) - \alpha\delta^{T+1}}, \quad V^p = \frac{\delta^T \frac{\pi^m}{2}(1-\alpha)}{1-\delta(1-\alpha) - \alpha\delta^{T+1}}$$

- $V^m \geq \pi^m(1-\alpha) + \delta V^P$

$$\Leftrightarrow (1-\delta^{T+1}) V^m \geq \pi^m (1-\alpha)$$

$$\Leftrightarrow \frac{(1-\delta^{T+1})}{2(1-\delta(1-\alpha)-\alpha\delta^{T+1})} \geq 1$$

$$\Leftrightarrow 1-\delta^{T+1} \geq 2 - 2\delta(1-\alpha) - 2\alpha\delta^{T+1}$$

$$\Leftrightarrow (1-2\alpha)(\delta - \delta^{T+1}) \geq 1-\delta$$

$$\Leftrightarrow (1-2\alpha)\delta(1-\delta)(1+\delta+\ldots+\delta^{T-1}) \geq 1-\delta$$

- True for T large and δ close to 1.
- When α small, can choose T small.

Observations

1. We get partial collusion here: $\pi^* < \pi^m$
2. Price wars occur in equilibrium
   - Price wars are part of a functioning cartel.
3. With a continuous noise specification, firms sometimes get away with cheating. (also need continuous demands across firms.)
4. Price wars are triggered by signals of cheating

Porter "A Study of Cartel Stability: The Joint Executive Committee, 1880-1886."

- Rob Porter's PhD dissertation
- Excellent empirical paper
- Collusion was legal until 1887.

- Question: are there price wars in the data?

Model:

- Demand: $\log Q_t = \alpha_0 - \alpha_1 \log P_t + \alpha_2 \text{lakes}_t + \tilde{u}_t \Rightarrow \varepsilon = -\alpha_1$
- Supply: $\frac{p - mc}{p} = -\frac{\theta}{\varepsilon}$ $\quad \theta = \begin{cases} 0 & \text{if mc pricing} \\ 1 & \text{if monopoly pricing} \\ 1/N & \text{if Cournot pricing} \end{cases}$

- assume $\theta = \begin{cases} \theta^c & \text{in collusive phase} \\ \theta^w & \text{in price war phase} \end{cases}$

- assume $mc_t = \eta_0 Q_t^{\beta_1} \tilde{v}_t$

$$\Rightarrow \frac{P_t - \eta_0 Q_t^{\beta_1} \tilde{v}_t}{P_t} = \frac{\theta}{\alpha_1}$$

$$\Rightarrow P_t\left(1 - \frac{\theta}{\alpha_1}\right) = \eta_0 Q_t^{\beta_1} \tilde{v}_t$$

$$\Rightarrow \log P_t = -\log\left(1 - \frac{\theta}{\alpha_1}\right) + \log \eta_0 + \beta_1 \log Q_t + \log \tilde{v}_t$$

Estimation:

1] $\log P_t = \beta_0 + \beta_1 \log Q_t + \beta_2 \underbrace{I_t}_{\text{collusion dummy}} + v_t$

$$\beta_0 = \log \eta_0 - \log\left(1 - \frac{\theta_w}{\alpha_1}\right)$$
$$\beta_2 = \log\left(1 - \frac{\theta_w}{\alpha}\right) - \log\left(1 - \frac{\theta_c}{\alpha}\right)$$

2] $\log Q_t = \alpha_0 - \alpha_1 \log P_t + \alpha_2 \text{lakes}_t + \hat{u}_t$

- η_0, θ_w, and θ_c are not separately identified.
  - Porter assumes $\theta^w = 0$.

<u>Estimation 1:</u> I_t comes from newspaper reports.

- standard instrumental variables estimation
  - I_t is an instrument for P_t in dmd equation
  - $lakes_t$ is an instrument for Q_t in supply equation
- Estimates: $\hat{\beta}_2 = 0.382$

  $\hat{\alpha}_1 = 0.742$

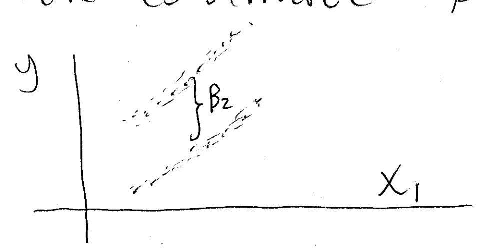
  $\theta_w = 0 \Rightarrow \theta_c = 0.24$

<u>Estimation 2:</u> Switching regressions model

- suppose you want to estimate:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

- but we have no data on x_2. Can we estimate this?
- assume $x_2 \in \{0, 1\}$ and $\varepsilon \sim N$. Then we can estimate β_2:
- random effects.



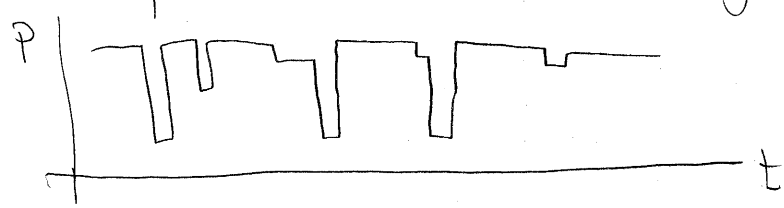
- estimate with v_t normal and I_t as the unobservable switching variable.

Results: ◦ $\hat{\beta}_2 = 0.545 \Rightarrow \hat{\theta}_c = 0.336$

◦ $\text{Prob}(I_t = 1) = 0.72$

◦ Fit is significantly better than the model without unobservable I_t.

(*) In problem set, do a graph of price vs time:



Addendum: Controlling for serial correlation, Ellison gets $\hat{\alpha}_1 = 1.802$, $\hat{\beta} = 0.637 \Rightarrow \hat{\theta}_c = 0.85$