- self-selection once more
- supply function equilibria
- simultaneous equations
- demand estimation
- repeated games

Self-selection

"virtual valuation" - what I get from type θ if I sell to this type.

$$\int_{\underline{\theta}}^{\bar{\theta}} [\overbrace{v(q(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q(\theta),\theta)} - cq(\theta)] f(\theta) d\theta$$

$\theta \sim F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$

- obviously, if the virtual valuation for a type is negative, we do not sell to this type
  - raise $\underline{\theta}$ to $\hat{\theta}$. Then sell to $[\hat{\theta}, \bar{\theta}]$ (truncate the support)
  - $$u_0 = \underbrace{\underline{u}}_{=0} + \underbrace{\int_{\underline{\theta}}^{\hat{\theta}} \underbrace{v_\theta dx}_{=0}}_{=0} + \int_{\hat{\theta}}^{\bar{\theta}} v_\theta dx$$
    $\rightarrow$ Less rents you have to leave to higher types
  - This is most likely to show up in auctions

Supply function equilibria

- 2 firms, symmetric
- $Q = D(p, \varepsilon)$ $\quad \varepsilon \sim F(\varepsilon)$ on $[\underline{\varepsilon}, \bar{\varepsilon}]$
- Firms submit supply functions $Q_i(p), Q_{-i}(p)$
- Market-maker chooses p^* s.t. $Q_i(p^*) + Q_{-i}(p^*) = D(p^*, \varepsilon)$
  - (Q, p, ε), Q_i, Q_{-i}, $\underline{P(Q_{-i}, Q_i, \varepsilon)}$ need to be known.

- Thus, firm i says: if ε happens, then $-i$ submits Q_{-i}, and market-maker chooses $P(\cdot)$. In response, I choose the monopoly quantity on the residual demand curve:

$$\max_p \{pQ_i - c(Q_i)\}$$

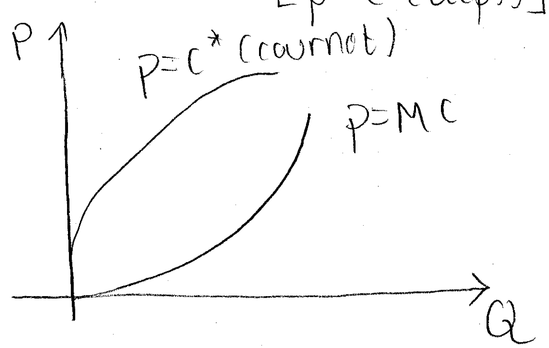
$$= \max_p \{p[D(p,\varepsilon) - Q_{-i}(p)] - c(D(p,\varepsilon) - Q_{-i}(p))\}$$

$$(p): \underbrace{[D(p,\varepsilon) - Q_{-i}(p)]}_{Q_i(p)} + (p-c)[D(p,\varepsilon) - Q_{-i}(p)][D_p(p,\varepsilon) - \underline{Q_{-i}'(p)}] = 0$$

this is the main difference

- Focusing on symmetric equilibria (ie $Q_i = Q_{-i}$) and horizontal demand shocks: $D_{p\varepsilon} = 0$, we have

$$Q'(p) = \frac{Q(p)}{[p - c'(Q(p))]} + D_p(p,\varepsilon)$$



The solution will be between Cournot and perfect competition.

- use boundary condition: $p \to +\infty \Rightarrow q \to +\infty$

- In one-shot games, $Q'(p) = 0$. (opponent cannot respond in the future if I cheat.)
- This result is distribution-independent.

Simultaneous Equations

$q_i = Y_i \alpha_i + \beta_{1i} p_i + \varepsilon_{1i}$ (demand)

$p_i = D_i \gamma_i + \beta_{2i} q_i + \varepsilon_{2i}$ (supply)

- Simultaneous equations bias. (OLS is biased)
- Let $X_i = [Y_i \;\; P_i]$. Then $q_i = X_i \delta + \varepsilon_{1i}$

$$\hat{\delta}_{OLS} = [X'X]^{-1} X' q$$
$$= \delta + \begin{bmatrix} Y'Y & Y'p \\ p'Y & p'p \end{bmatrix} \begin{bmatrix} Y'\varepsilon_1 \\ p'\varepsilon_1 \end{bmatrix}$$

- consistent if $\text{plim} \frac{Y'\varepsilon_1}{N} = 0$ and $\text{plim} \frac{p'\varepsilon_1}{N} = 0$

But $p(q(\varepsilon))$, so the second one is not likely to hold.

I/O typically does IV on the demand side. We don't believe in the linear functional form of the supply side, typically.

Let $W = [Y \;\; D]$. Need: • $\text{plim} \frac{W'X}{N} = \Sigma_{WX}$ finite, nonsingular

$X = [Y \;\; P]$

- $\text{plim} \frac{W'\varepsilon_1}{N} = 0$
- $\text{plim} \frac{W'W}{N} = \Sigma_{WW}$ positive definite

$$\hat{\delta}_{IV} = [W'X]^{-1} W'q$$
$$= (W'X)^{-1} W'(X\delta + \varepsilon_1)$$
$$= \delta + (W'X)^{-1} W'\varepsilon_1$$
$$\Rightarrow \text{plim } \hat{\delta}_{IV} = \delta + \underbrace{\left(\text{plim } \frac{W'X}{N}\right)^{-1}}_{= \Sigma_{WX}} \underbrace{\text{plim } \frac{W'\varepsilon_1}{N}}_{=0} = \delta$$

Looking for instruments

- Valid instruments
  - uncorrelated with error terms
  - correlated with endogenous variables
  - don't enter the equation directly

Demand systems

i) Input prices

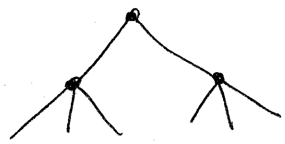
ii) prices of other products
  - random coefficients:
    mkt share = f (utility (price paid))

iii) prices of the product in different markets
  - (Hausman instrument)

iv) characteristics of other products
  - rarely enough variation in characteristic space

Demand Estimation

- N goods → N demand functions → N^2 elasticities
- Logit (many fcnal form assumptions)
- Nested logit: • choice hierarchy



- Principals of differentiation GEV: estimate a matrix.

Most used today: (very data-intensive)

- almost ideal demand system (Hausman likes this.)
- Random coefficients logit

$$u_{ijt} = x_{jt}\beta_i^* + \alpha_i^* p_j + \xi_{jt} + \varepsilon_{ij}$$

(β_i, α_i are individual-specific)

$$\begin{bmatrix} \alpha_i^* \\ \beta_i^* \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \underbrace{\Pi D_i}_{\text{observable demographics}} + \underbrace{\overset{\text{estimate the distribution of unobservables.}}{\Sigma} v_i}_{\text{unobservables}}$$

$$u_{i0t} = \xi_0 + \Pi_0 D_i + \sigma_0 v_{i0} + \varepsilon_{i0t} \quad \text{(not buy)}$$

$$\Rightarrow u_{ijt} = (x_j + \beta + \alpha p_{jt} + \xi_{jt}) + \underbrace{(\mu_{ijt} + \varepsilon_{ijt})}_{\text{ind. shock}}$$

δ_{jt} = "mean utility"

$$\mu_{ijt} = \begin{pmatrix} p_{jt} \\ y_{jt} \end{pmatrix} (\Pi D_j + \Sigma v_i)$$

Define the set of consumers who purchase product j:

$$A_{jt}(x_{\cdot t}, p_{\cdot t}, \delta_{\cdot t}; \theta_2) = \{(D_i, v_i, \varepsilon_{it}) : u_{ijt} \geq u_{ilt} \ \forall l = j\}$$

$$\underbrace{S_{jt}}_{\text{mkt shares}}(x_{\cdot t}, p_{\cdot t}, \delta_{\cdot t}; \theta_2) = \underbrace{\int_{A_{jt}} dP^*(D, v, \varepsilon)}_{\text{solve this numerically}}$$

θ_1 = linear parameter

θ_2 = non-linear

Repeated games

A \ B	C	D
C	x, x	0, y
D	y, 0	z, z

$y > x > z > 0$

$2x > y$

Use trigger strategy: C unless anyone has ever chosen D. Then choose D forever.

If C: $x + \delta x + \delta^2 x + \ldots = x + \frac{\delta}{1-\delta} x$

If D: $y + \delta z + \delta^2 z + \ldots = y + \frac{\delta}{1-\delta} z$

Cooperate if $x + \frac{\delta}{1-\delta} x \geq y + \frac{\delta}{1-\delta} z$

$$\Leftrightarrow \underbrace{\frac{\delta}{1-\delta}(x-z)}_{\text{future losses}} \geq \underbrace{y-x}_{\text{gains today}}$$